Energetic particle diagnostics

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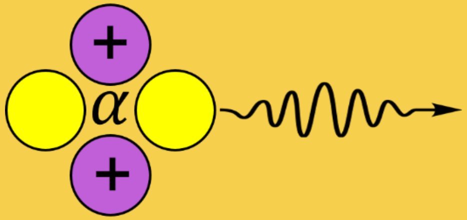
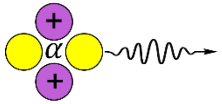


Outline

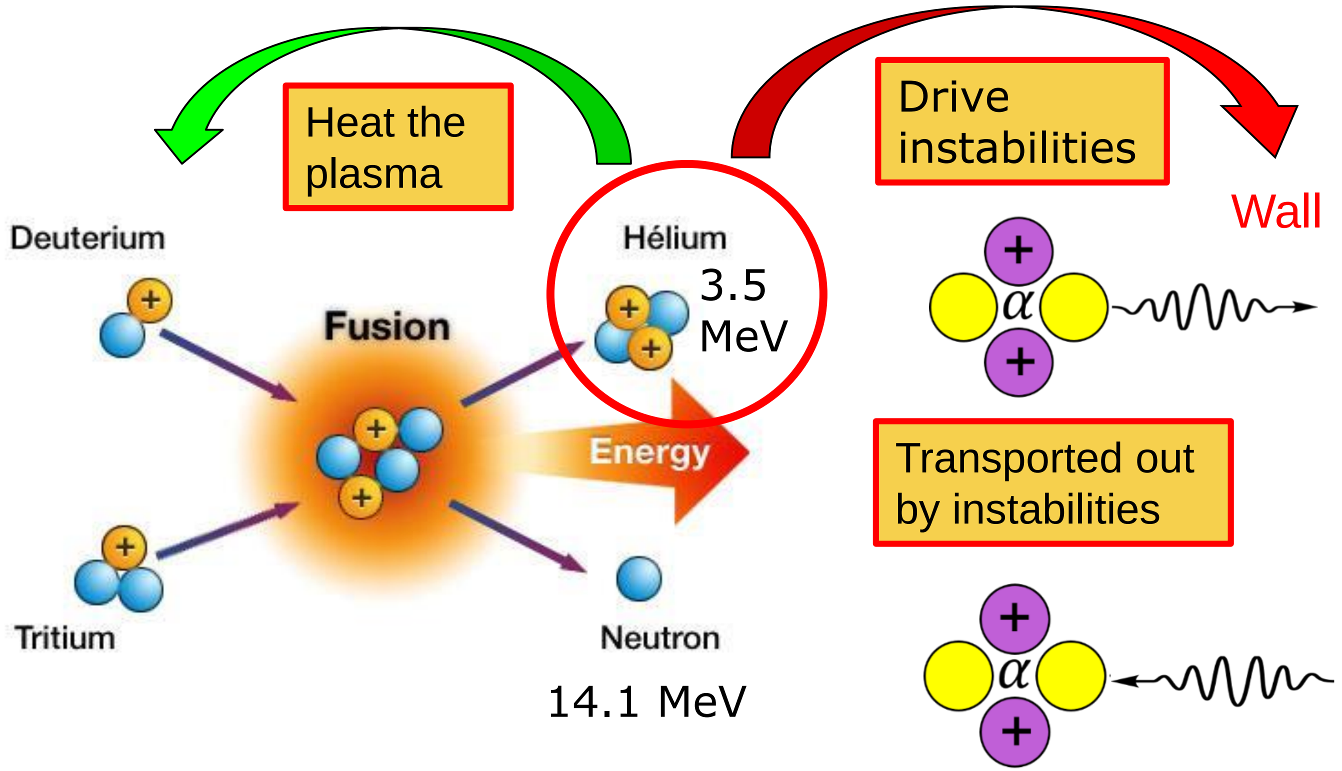


- **Introduction: Why diagnose energetic particles?**
- Physics of energetic particle diagnostics
- The forward problem: Spectrum formation for energetic particle diagnostics
- The inverse problem: Inferring energetic particle distributions from diagnostic data
- Summary

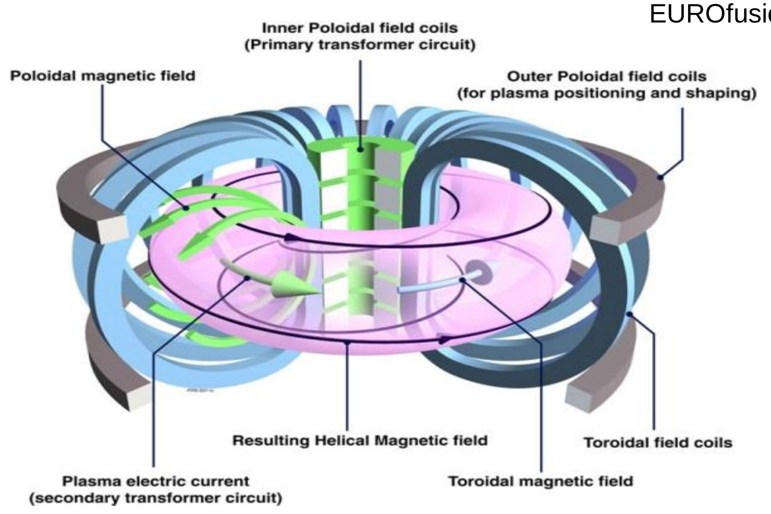
Energetic particles in a fusion plasma

- Helium born at 3.5 MeV 9×10^6 m/s, 3% of light speed
- Confined in a tokamak by strong magnetic fields
- Heat the plasma by collisions

$$\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$$



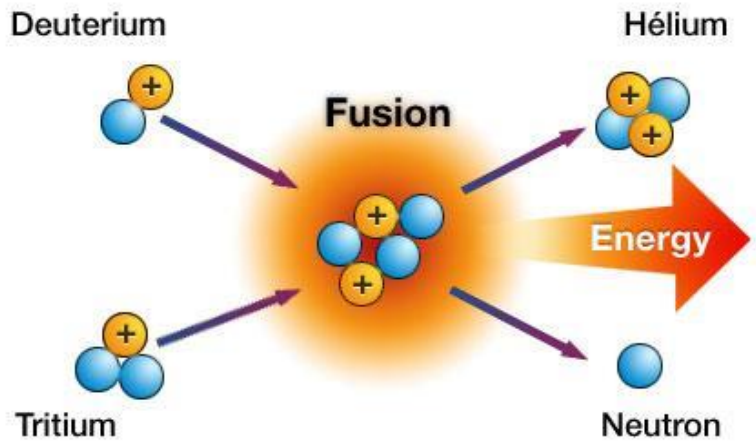
EUROfusion



Energetic particles

- Heat the plasma by collisions
- Drive instabilities
- Transported out by instabilities

Why does the α-particle get 3.5 MeV and the neutron 14.1 MeV ?



3.5 MeV

14.1 MeV

The $\alpha + n$ are a little lighter than $D + T$. The mass defect gives the reaction energy

$$E = \Delta m\, c^2 = 17.6\, MeV.$$

Why does the α-particle get 3.5 MeV and the neutron 14.1 MeV?

Why don't they get, e.g., half each?

Energetic particle distributions in tokamak plasmas

- **6D phase-space distribution function**

$f(\boldsymbol{x}, \boldsymbol{v})$

At every point in position space (3D), a velocity distribution function (3D).

- **4D phase-space distribution function**

$f(R, z, v_\parallel, v_\perp)$ or $f(R, z, E, p)$

Tokamak donut symmetry (2D position space), fast gyration (2D velocity space).

- **3D phase-space distribution function**

$f(E, \mu, P_\Phi, \sigma)$

constants of motion (energy, magnetic moment, canonical toroidal angular momentum, $\sigma = \pm 1$).

- **1D phase-space distribution function**

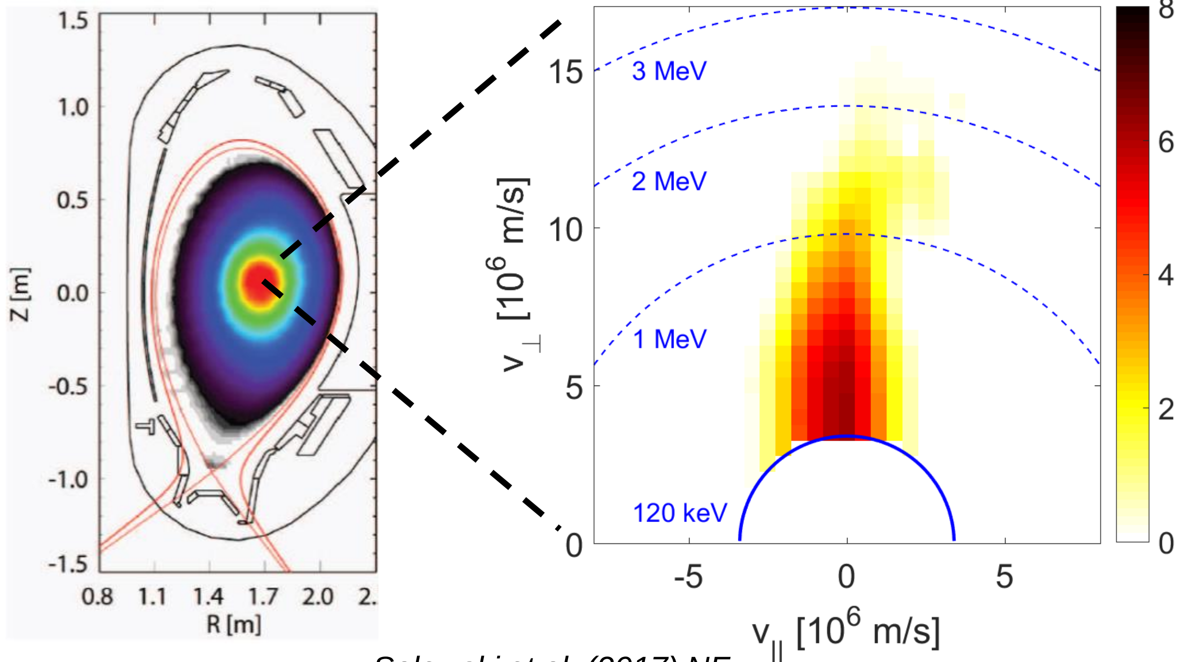
$f(E), f(v), f(v_\perp)$

2D position-space distribution function
(“fast-ion density profile”)

2D velocity-space distribution function
(in a tiny volume)

$$n(R,z) = \iint f(R, z, v_\parallel, v_\perp)\, dv_\parallel dv_\perp$$

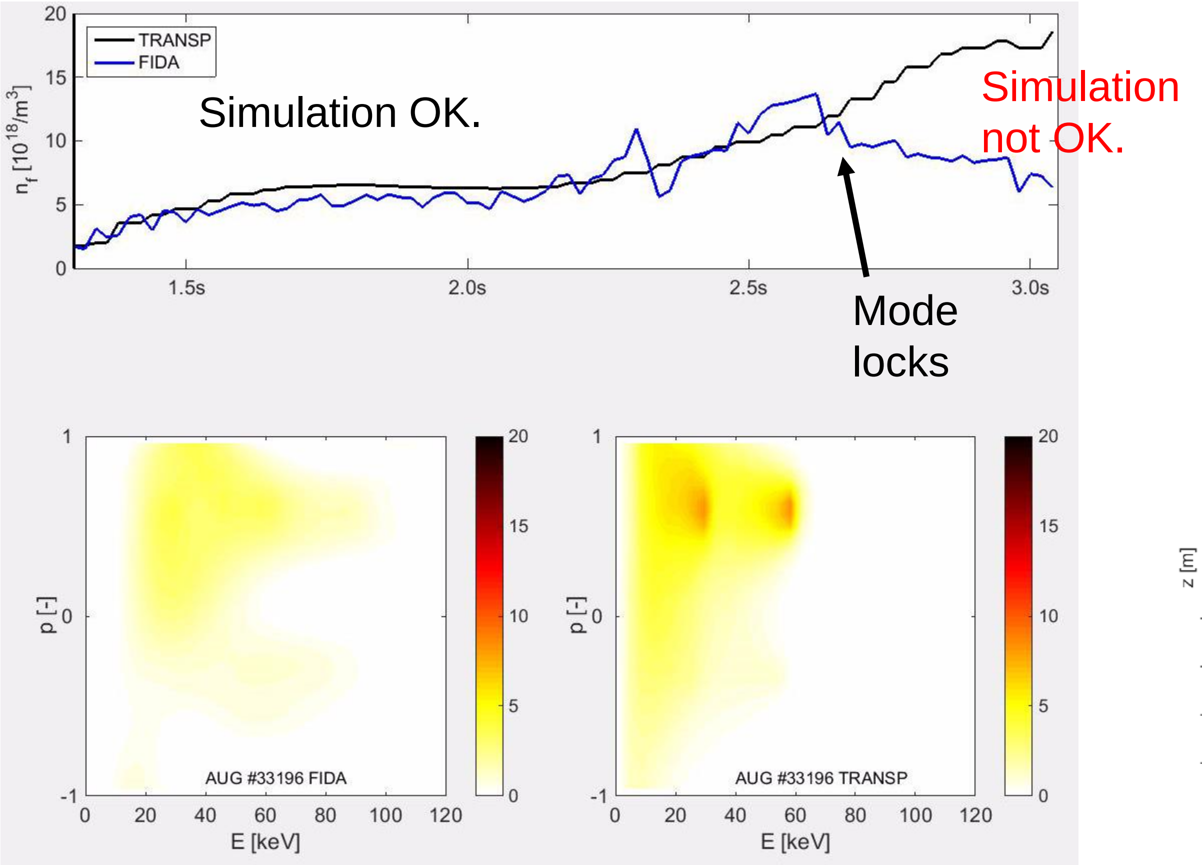
$$f(R_0, z_0, v_\parallel, v_\perp)$$



Salewski et al. (2017) NF

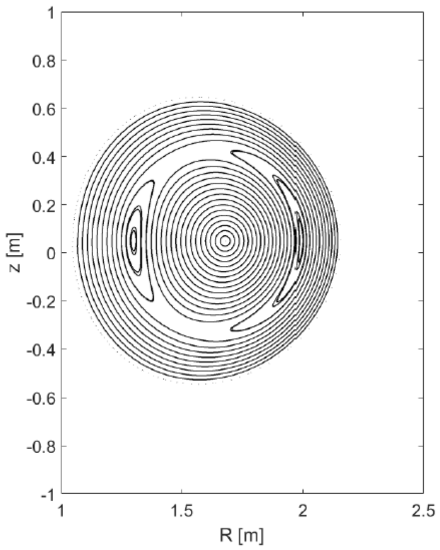
Why diagnose energetic particles? To check theory!

Energetic particle density n_{fast} $n = \iint f \, dv_\parallel dv_\perp$



Energetic particle velocity distribution function f_{fast}

- 2/1 neoclassical tearing mode (NTM)
- Simulation and measurement agree when the mode rotates
- Central fast-ion density decreases when the mode locks



Relation between $(v_\parallel, v_\perp)$ coordinates and (E, p) (energy, pitch) coordinates

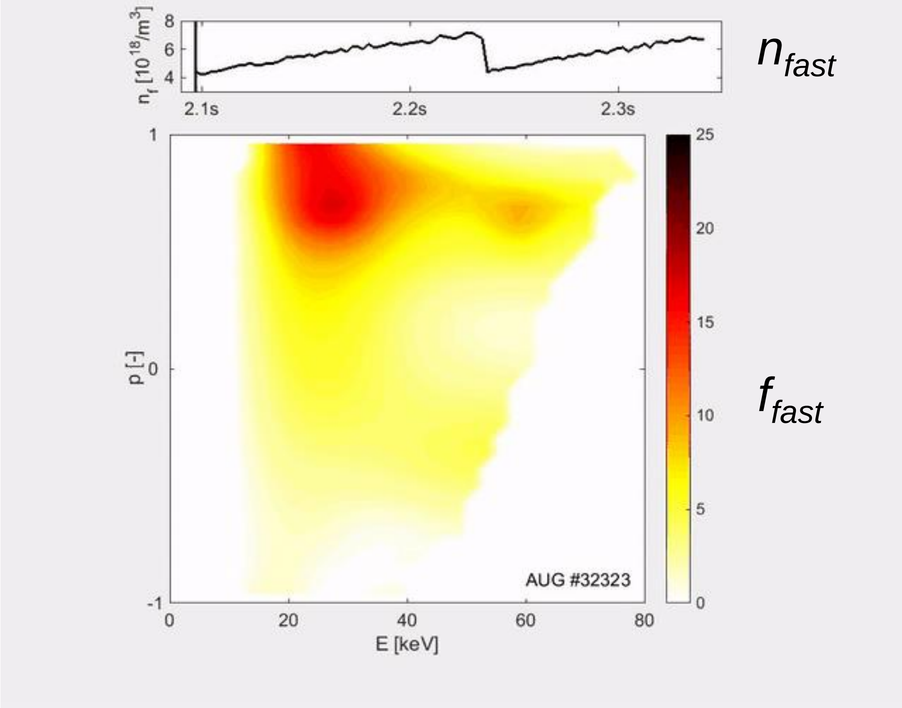
$$E = \frac{1}{2} m v^2 \qquad p = \frac{v_\parallel}{v}$$

Usually p>0 is in the direction of the current, not ***B***.

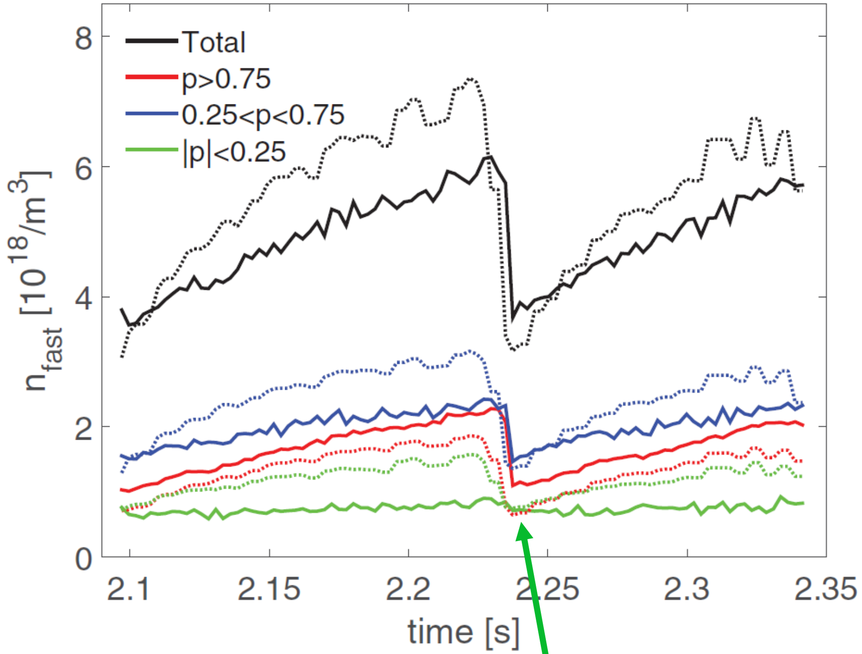
Why diagnose energetic particles? To check theory!

Sawtooth instability

- Periodic violent bursts ejecting particles and energy from the plasma core
- Time traces of *T, n, p,* n_{fast} look like a sawtooth



$$n_{fast} = \iint f \, dv_{\parallel} dv_{\perp}$$



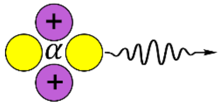
- Measured and simulated fast-ion densities for a sawtooth crash (TRANSP Kadomstev model)

TRANSP sawtooth in n_{fast} for $|p|<0.25$ but measurement flatlines!

Salewski et al. 2016 NF

Outline

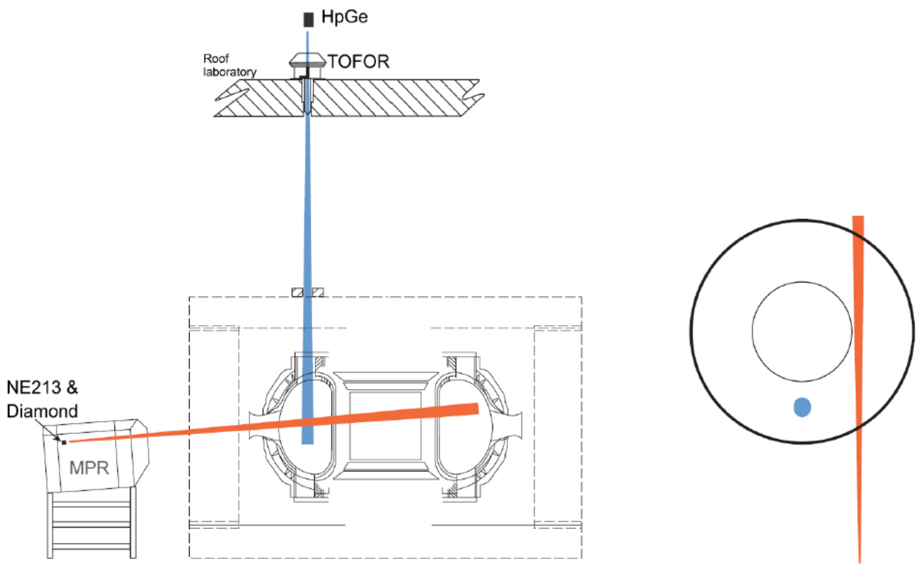
- Introduction: Why diagnose energetic particles?



- **Physics of energetic particle diagnostics**
- The forward problem: Spectrum formation for energetic particle diagnostics
- The inverse problem: Inferring energetic particle distributions from diagnostic data
- Summary

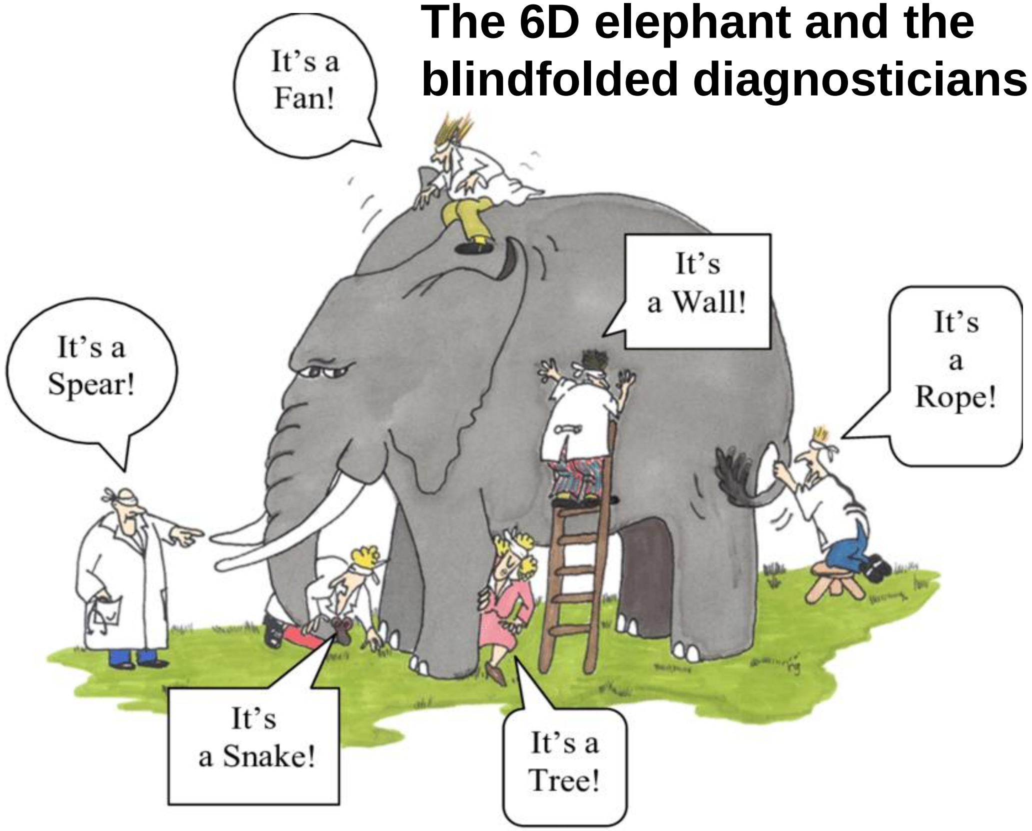
How to diagnose a 6D phase-space distribution function?

- Each diagnostic signal on a tokamak probes a tiny, different part of 6D phase space
- In donut-symmetric plasma and for axisymmetric velocity distributions (4D), the diagnostics cover a lot more, but usually still only a small part.

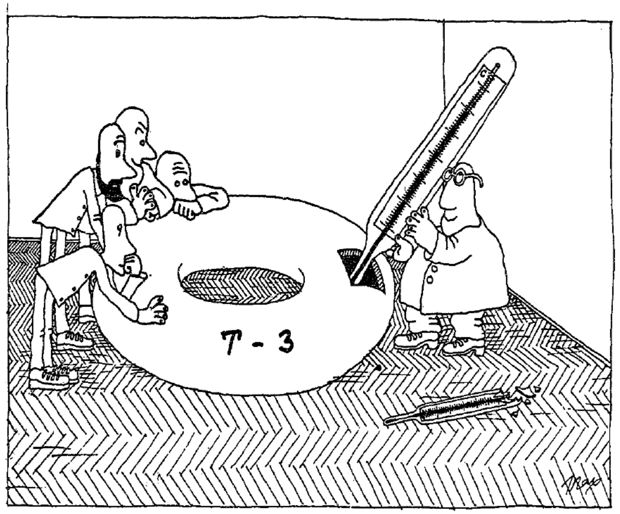


Eriksson et al. 2015 NF, Salewski et al. 2017 NF

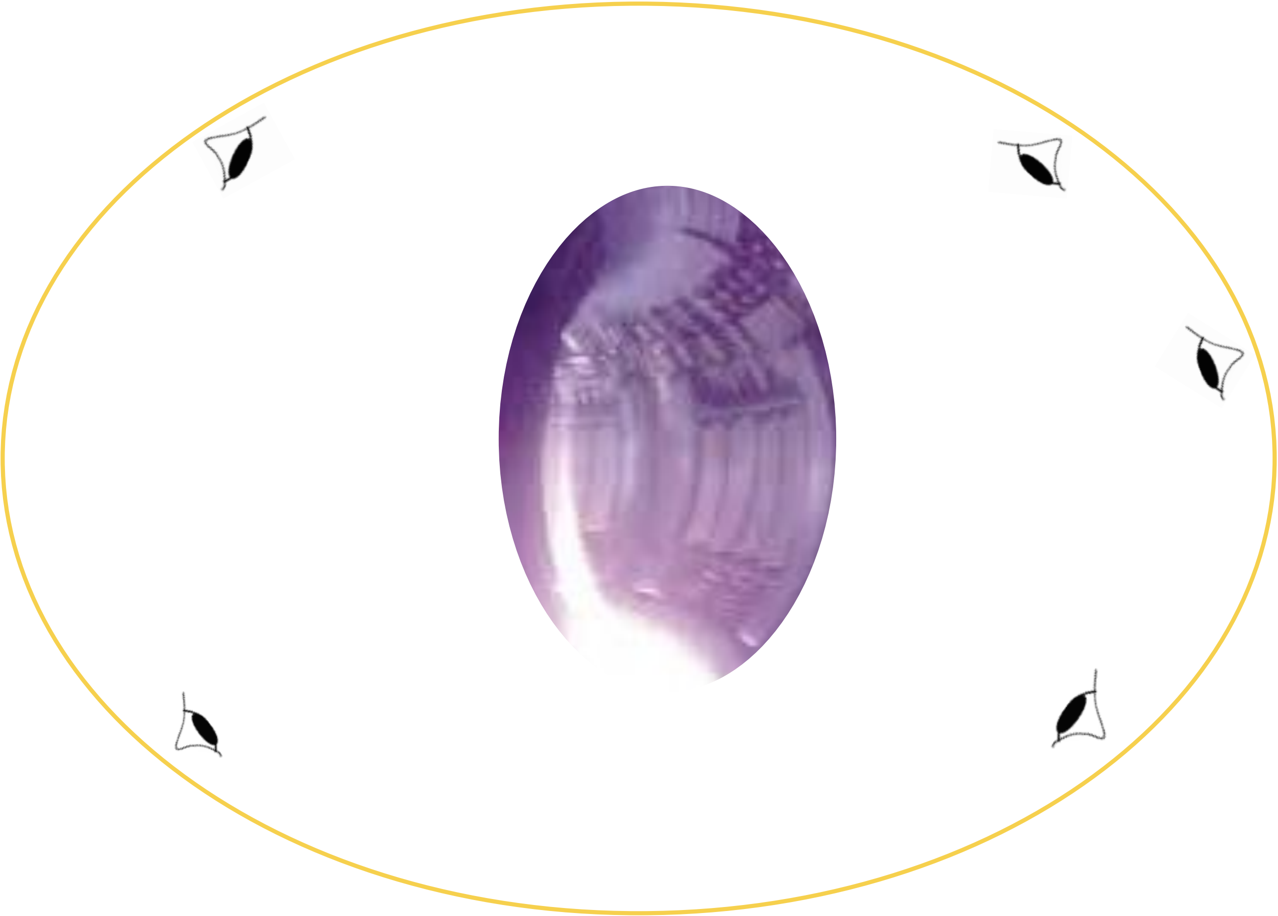
The 6D elephant and the blindfolded diagnosticians



What leaves the plasma and can be detected?



Kadomtsev (~1968)



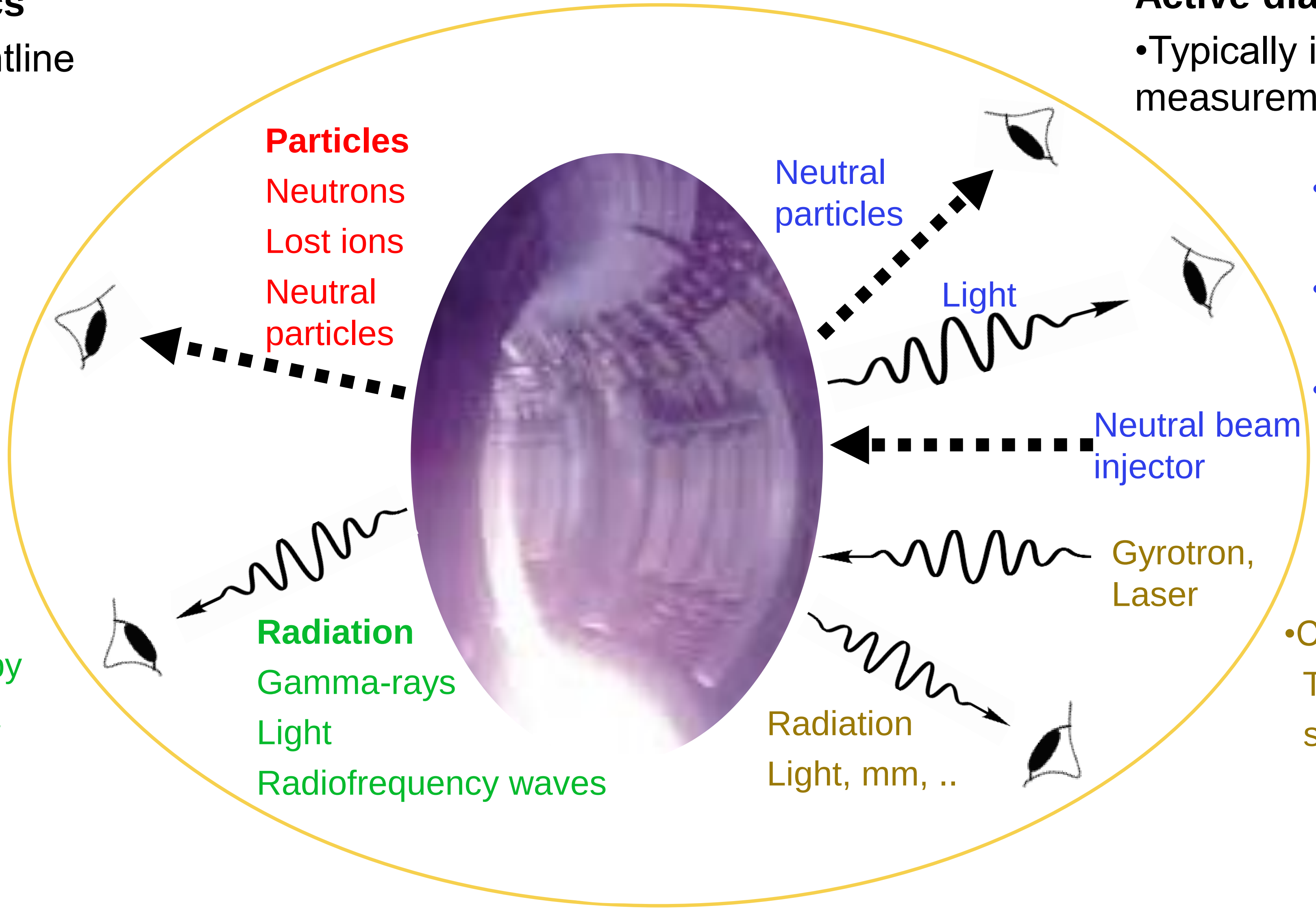
Energetic particle diagnostics

Passive diagnostics

•Typically along sightline

- •Neutron counter
- •Neutron camera
- •Neutron emission spectroscopy
- •Fast-ion loss detector
- •Charged fusion product detector
- •Passive neutral particle analyser

- •Gamma-ray camera,
- •Gamma-ray spectroscopy
- •Passive fast-ion D-alpha spectroscopy
- •Ion cyclotron emission spectroscopy



Active diagnostics

•Typically in a small measurement volume

- •Neutral particle analyser
- •Imaging neutral particle analyzer
- •Fast-ion D-alpha spectroscopy

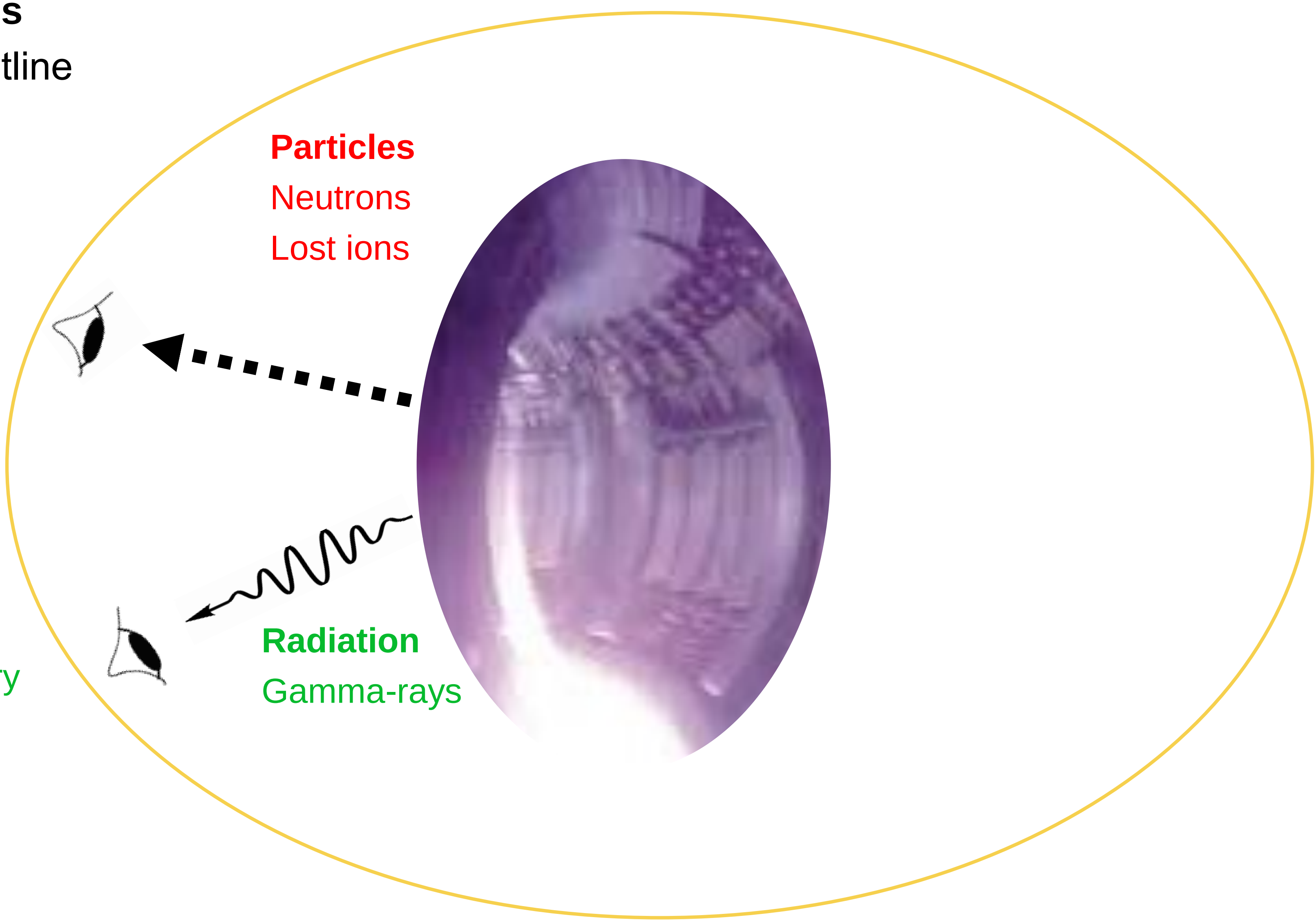
- •Collective Thomson scattering

Energetic particle diagnostics: Fusion products

Passive diagnostics

•Typically along sightline

- Neutron counter
- Neutron camera
- Neutron emission spectroscopy
- Charged fusion product detector

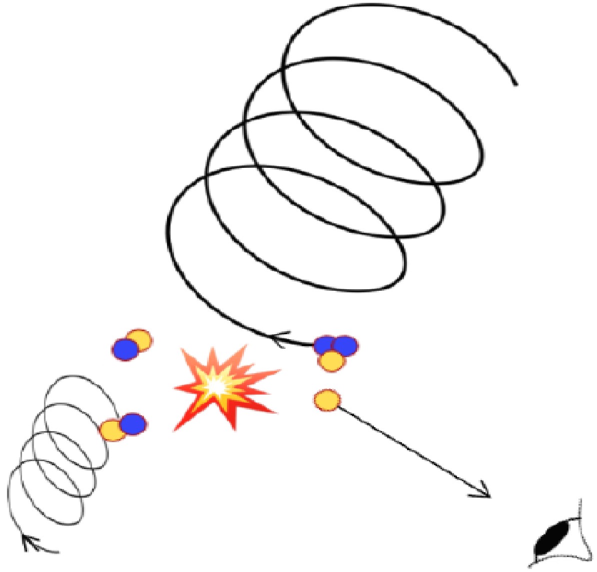


- Gamma-ray camera,
- Gamma-ray spectrometry

Fusion product diagnostics: neutrons and γ-rays

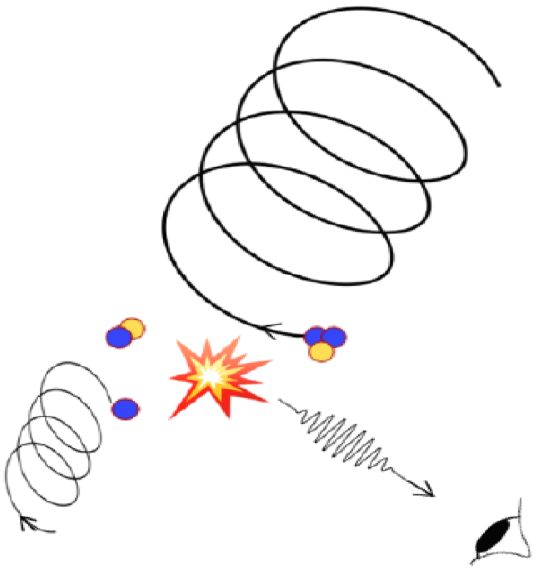
Neutron reactions

$$D + T \rightarrow \alpha\,(3.5\,MeV) + n\,(14.1\,MeV)$$



γ-ray one-step reactions

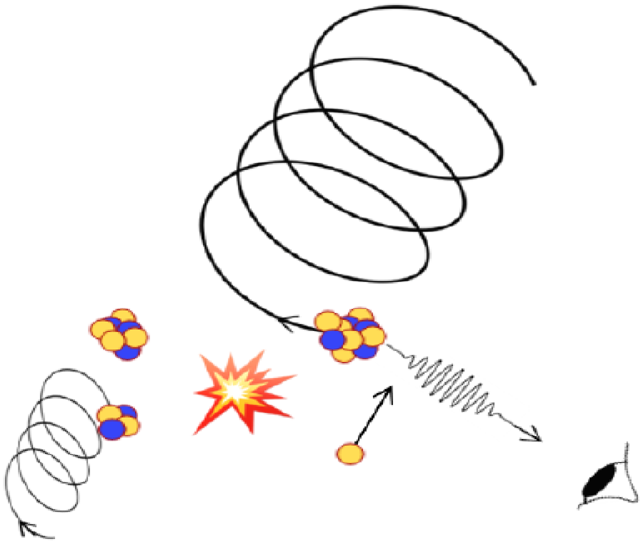
$$D + T \rightarrow {}^{5}He + \gamma\,(16.7\,MeV)$$



γ-ray two-step reactions

$$\alpha + {}^{9}Be \rightarrow n + {}^{12}C^{*(1,2)}$$

$${}^{12}C^{*(1)} \rightarrow {}^{12}C + \gamma\,(4.44\,MeV)$$



Salewski (2020) dr. thesis

Fusion product diagnostics: example neutron and γ-ray reactions

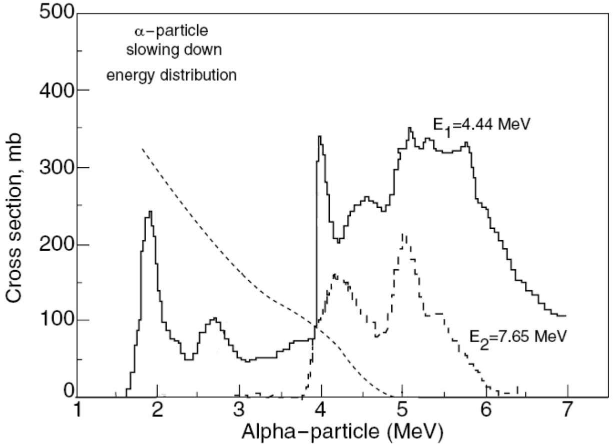
Neutron fusion reactions	γ-ray one-step fusion reactions	γ-ray two-step fusion reactions • De-excitation of excited nuclei	
Examples for contemporary high-performance plasmas			
• 2.45 MeV neutrons $D + D \rightarrow {}^{3}He + n$	$D + D \rightarrow {}^{4}He + \gamma$ $D + p \rightarrow {}^{3}He + \gamma$	$D + {}^{12}C \rightarrow p + {}^{13}C^{*(1)}$ ${}^{13}C^{*(1)} \rightarrow {}^{13}C + \gamma\ (3.09\ MeV)$ $D + {}^{9}Be \rightarrow n + {}^{10}B^{*}$ ${}^{10}B^{*} \rightarrow {}^{10}B + \gamma\ (2.868\ MeV)$	$D + {}^{9}Be \rightarrow p + {}^{10}Be^{*}$ ${}^{10}Be^{*} \rightarrow {}^{10}Be + \gamma\ (3.367\ MeV)$
Examples for burning or weakly burning fusion plasmas			
• 14.1 MeV neutrons $D + T \rightarrow {}^{4}He + n$	• Weak branches but observed in DT $D + T \rightarrow {}^{5}He + \gamma\ (\sim 13.5\ MeV)$ $D + T \rightarrow {}^{5}He + \gamma\ (16.7\ MeV)$ $T + p \rightarrow {}^{4}He + \gamma$	• Beryllium walls $\alpha + {}^{9}Be \rightarrow n + {}^{12}C^{*(1,2)}$ ${}^{12}C^{*(1)} \rightarrow {}^{12}C + \gamma\ (4.44\ MeV)$ ${}^{12}C^{*(2)} \rightarrow {}^{12}C^{*(1)} + \gamma\ (3.21\ MeV)$	• Boronization, pellets $\alpha + {}^{10}B \rightarrow p + {}^{13}C^{*(1,2,3)}$ ${}^{13}C^{*(1)} \rightarrow {}^{13}C + \gamma\ (3.09\ MeV)$ ${}^{13}C^{*(2)} \rightarrow {}^{13}C + \gamma\ (3.68\ MeV)$ ${}^{13}C^{*(3)} \rightarrow {}^{13}C + \gamma\ (3.85\ MeV)$

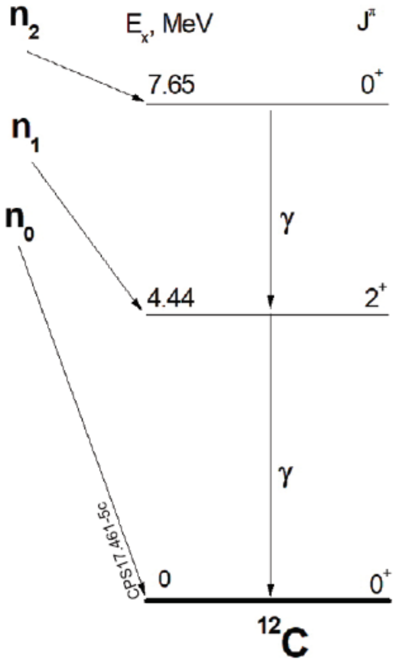
Typically >0.5 MeV needed for significant γ-ray production

γ-ray spectroscopy (GRS) – two-step reactions and excited nuclei

- Beryllium walls

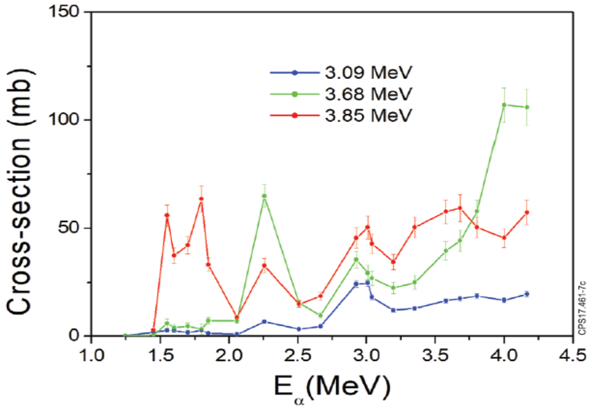
$$\alpha + {}^{9}Be \rightarrow n + {}^{12}C^{*(1,2)}$$
$${}^{12}C^{*(1)} \rightarrow {}^{12}C + \gamma\ (4.44\ MeV)$$
$${}^{12}C^{*(2)} \rightarrow {}^{12}C^{*(1)} + \gamma\ (3.21\ MeV)$$

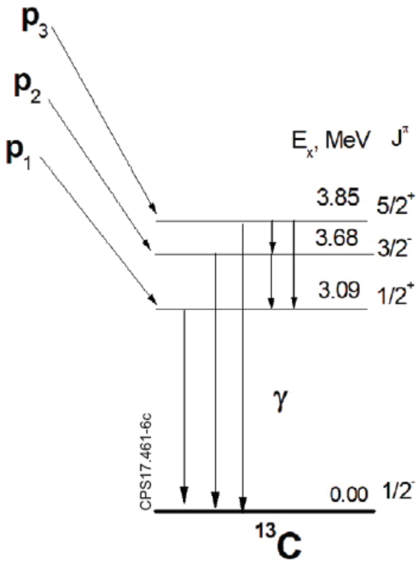




- Boronization, pellets

$$\alpha + {}^{10}B \rightarrow p + {}^{13}C^{*(1,2,3)}$$
$${}^{13}C^{*(1)} \rightarrow {}^{13}C + \gamma\ (3.09\ MeV)$$
$${}^{13}C^{*(2)} \rightarrow {}^{13}C + \gamma\ (3.68\ MeV)$$
$${}^{13}C^{*(3)} \rightarrow {}^{13}C + \gamma\ (3.85\ MeV)$$

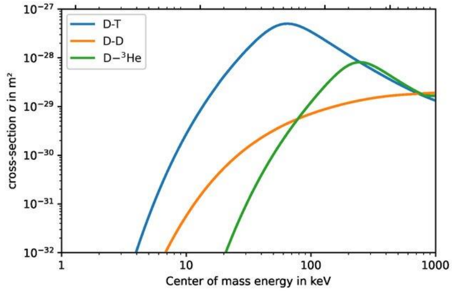


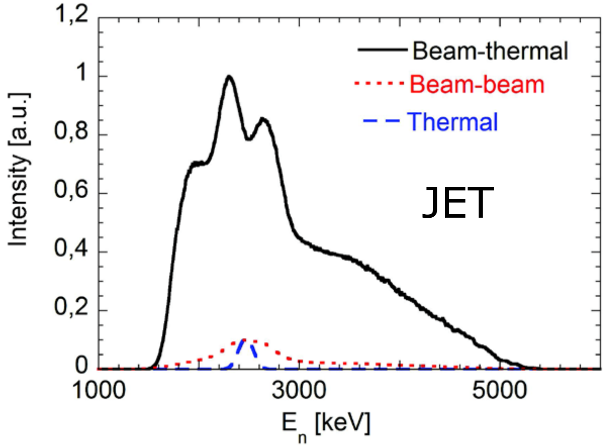


Kiptily (2018) NF

Nuclear reactions: thermonuclear, beam-target and beam-beam, and neutron counters

$$D + D \rightarrow {}^{3}He\ (0.82\ MeV) + n\ (2.45\ MeV)$$





Salewski et al. (2017) NF

***Thermonuclear** :*

- *Dominates in burning plasmas*

$$\dot{n}_n \propto P_{th} = \frac{1}{2} n_{D,th}^2 < \sigma v >_{th} E_{DD}$$

Beam-target:

- Often dominates in contemporary plasmas

$$\dot{n}_n \propto P_{beam-target} = n_{D,beam}\, n_{D,th} < \sigma v >_{beam} E_{DD}$$

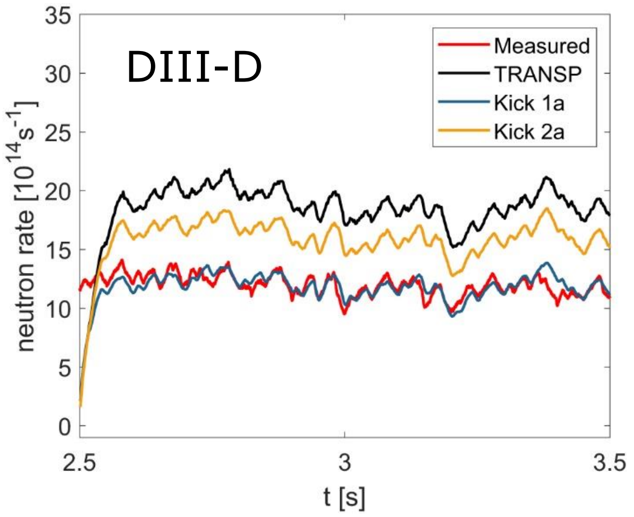
Beam-beam:

- Often less important, since

$$n_{D,beam} \ll n_{D,th}$$

$$\dot{n}_n \propto P_{beam-beam} = n_{D,beam}^2 < \sigma v >_{beam} E_{DD}$$

Neutron counter



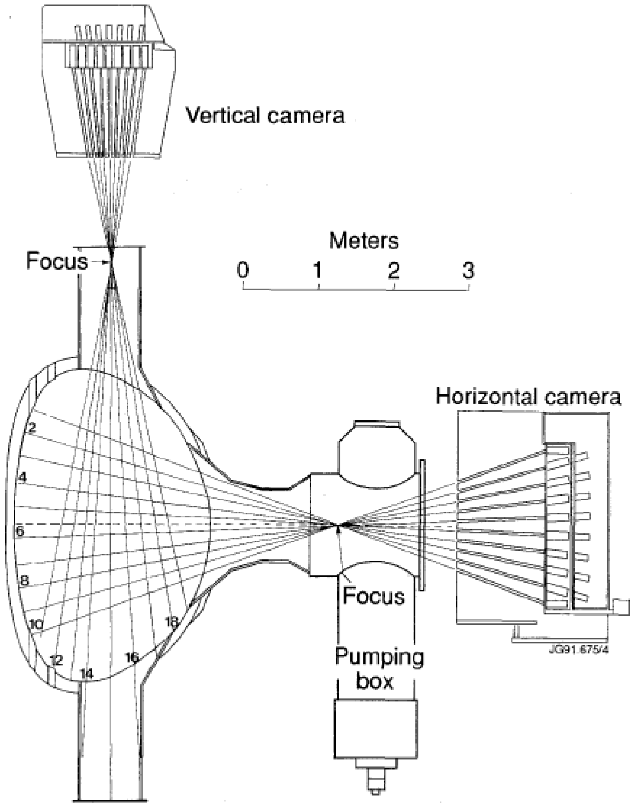
Madsen et al. (2020) NF

Beam-target dominated plasmas (common in contemporary tokamaks):

Neutron rate		Fast-ion density
$\dot{n}_n$	$\underset{\sim}{\propto}$	$n_{D,beam}$

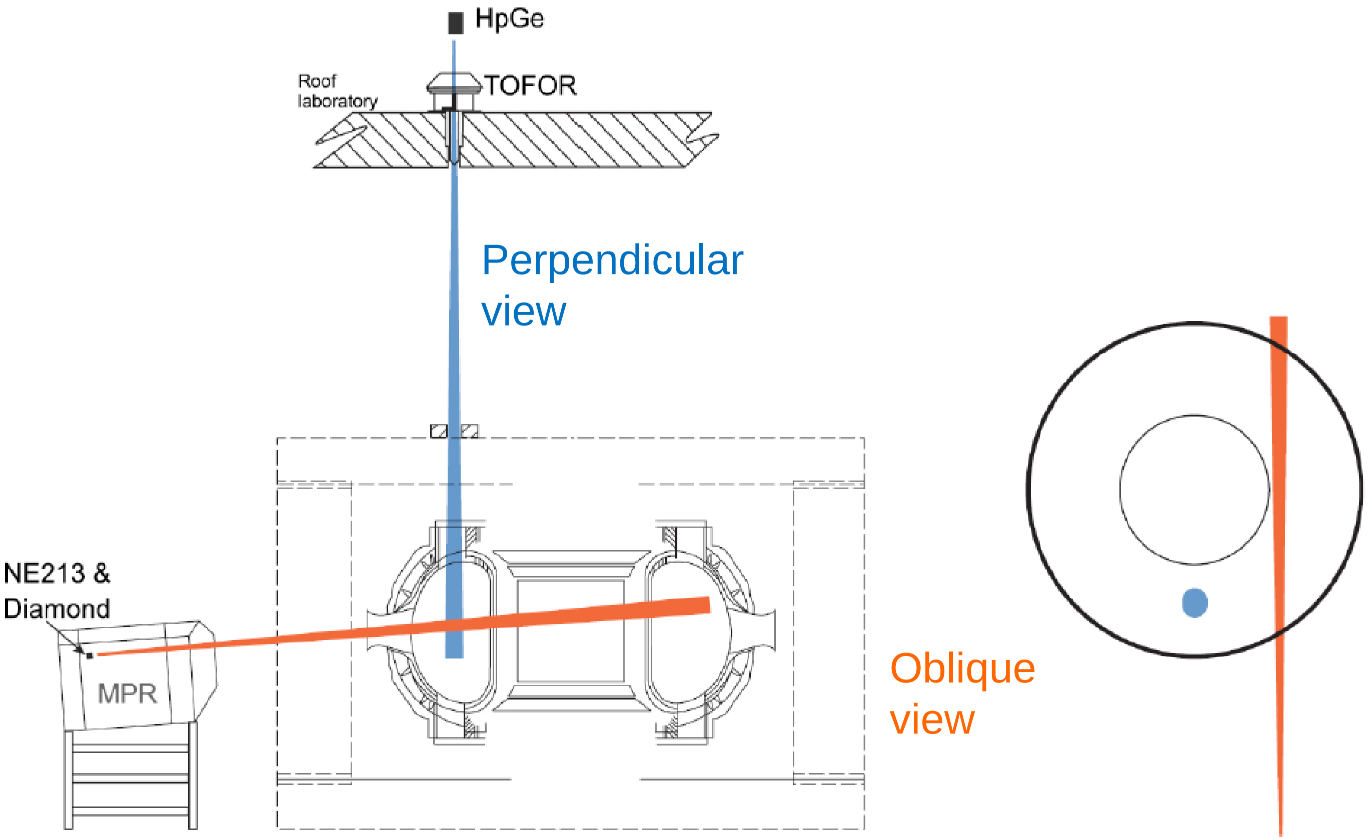
Neutron and γ-ray diagnostics at JET

Neutron and γ-ray cameras



Jarvis et al. (1997) FED

Neutron and γ-ray emission spectroscopy (NES and GRS)



Eriksson et al. (2015) NF, Salewski et al. (2017) NF

Neutron and γ-ray cameras at JET

- Count neutrons or γ-rays
- 9 vertical sightlines
- 10 horizontal sightlines
- Allows tomographic reconstruction

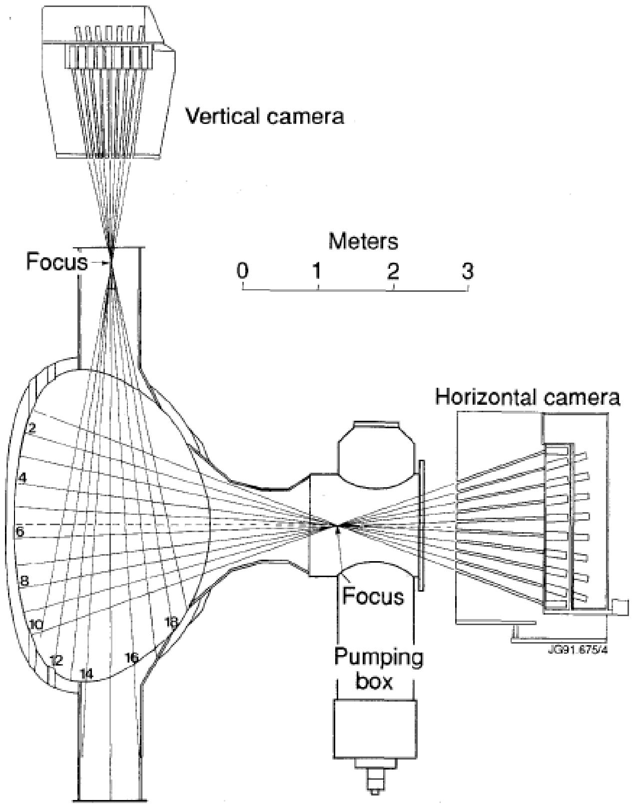
Neutron detectors

- NE213 liquid scintillator (2.5 & 14 MeV)
- Bicron-418 plastic scintillator (14 MeV)

γ-ray detectors:

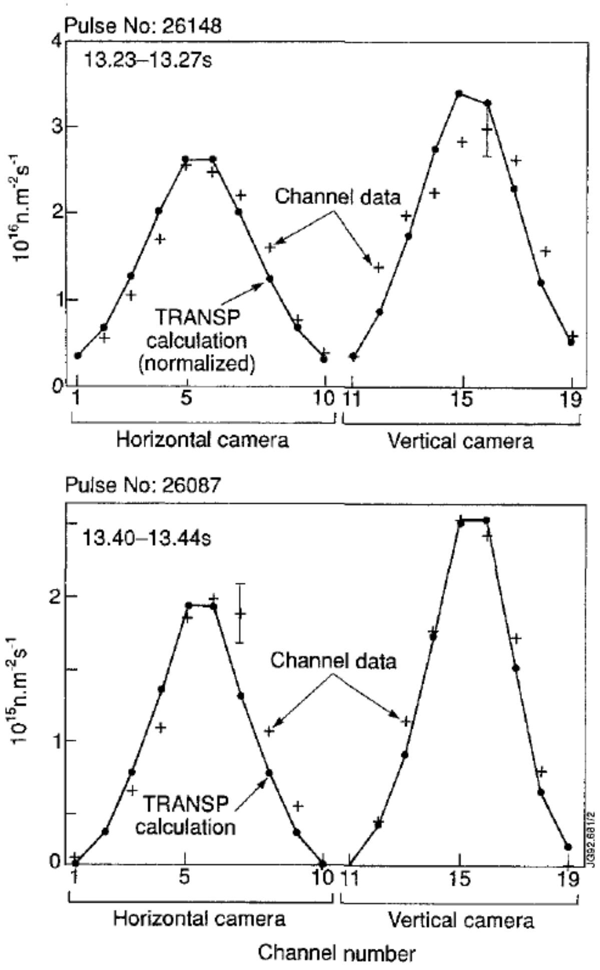
- Fast scintillators (~20ns decay times)

´
  - LaBr3
  - CeBr3

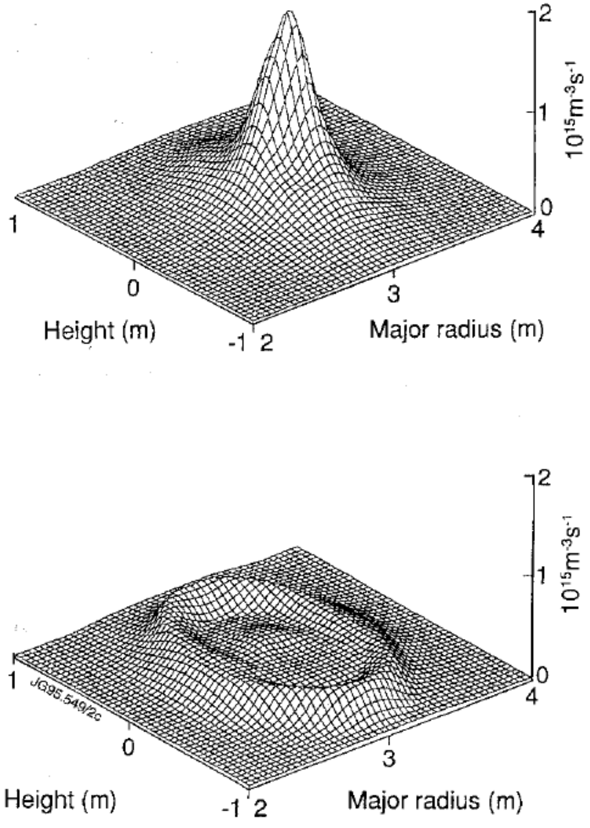


Jarvis et al. (1997) FED

Neutron camera at JET

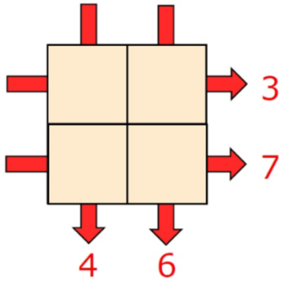


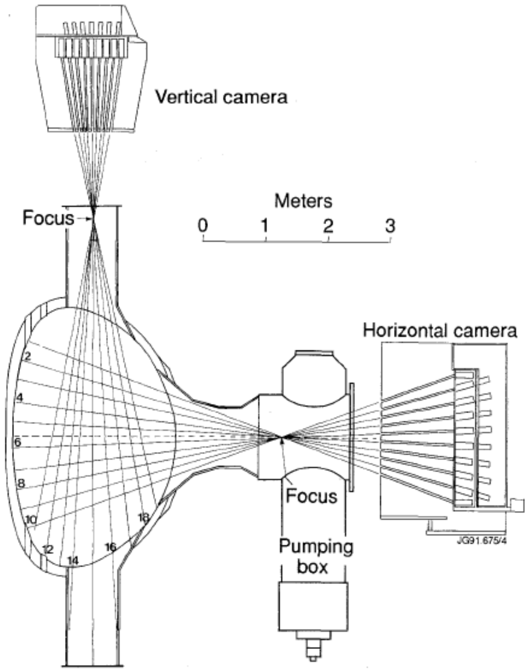
Measurement and TRANSP simulation



Before and after a sawtooth crash

Our back-of-the-envelope homework problem is analogous to the neutron camera tomography problem

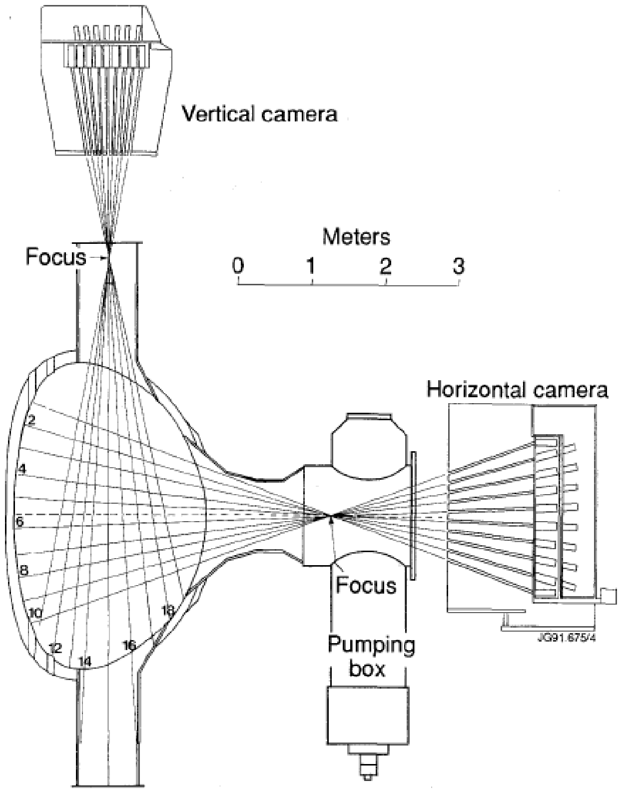




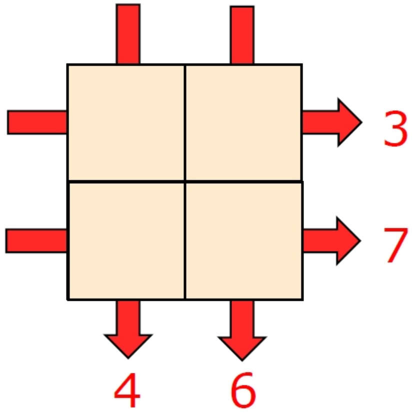
Jarvis et al. (1997) FED

Back-of-the-envelope neutron tomography problem

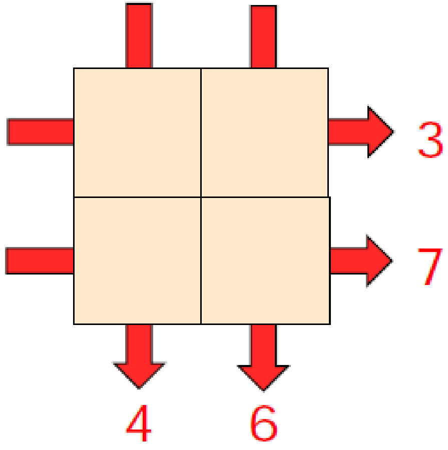
Often in tomography problems, one can measure the sum of emitted signal along a ray through a 2D plane. The ray path is given by the line-of-sight of a detector. The signal could be the count rate of neutrons generated in fusion reactions. Let's find which of the 4 quadrants below most likely emitted the neutrons, given 4 detectors measuring the row sums and the column sums. Assume that each square emits isotropically, i.e. the emission in all directions is the same. The counting rate is 3,7,4 and 6 neutrons per unit time in the four detectors.



Now let's formulate this as a mathematical problem: Determine the four unknown elements of the 2x2 matrix from the row and column sums along the orange arrows.

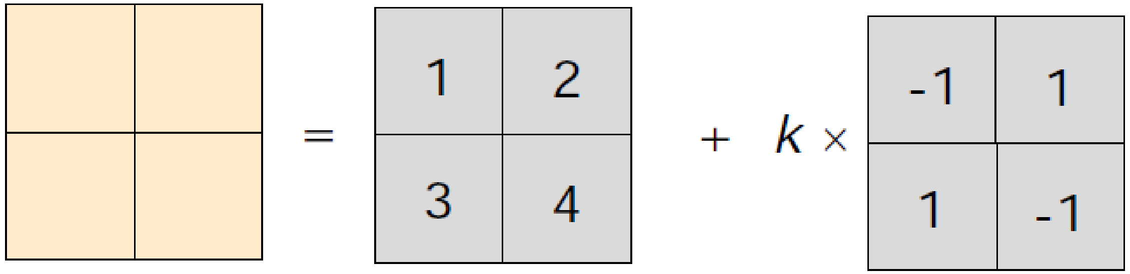


Back-of-the-envelope neutron tomography problem



$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \\ 4 \\ 6 \end{pmatrix}$$

Infinitely many solutions ($k \in \Re$):

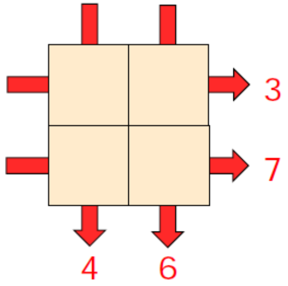


- This 4x4 matrix has only rank 3 (row 1 + row 2 – row 3 = row 4)
- Rank-deficient matrices are typical for tomography.
- Either no solution or infinitely many solutions
- With measurement noise: no solution, so we need to find a best-fit solution

P.C. Hansen

Back-of-the-envelope neutron tomography problem with prior information

Prior 1: Neutrons can’t be split and we can’t have a negative number of neutrons emitted, so the solution is integer and non-negative.



$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \\ 4 \\ 6 \end{pmatrix}$$

Infinitely many solutions ($k \in \Re$):

$$\begin{pmatrix} \; & \; \\ \; & \; \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + k \times \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$$

Prior: solution is integer and non-negative →

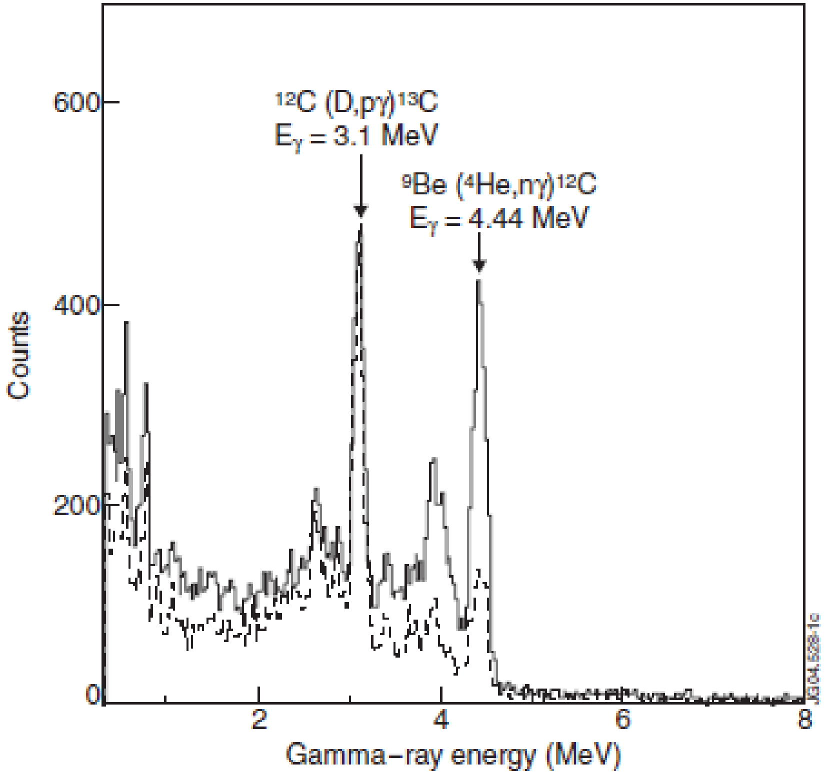
P.C. Hansen

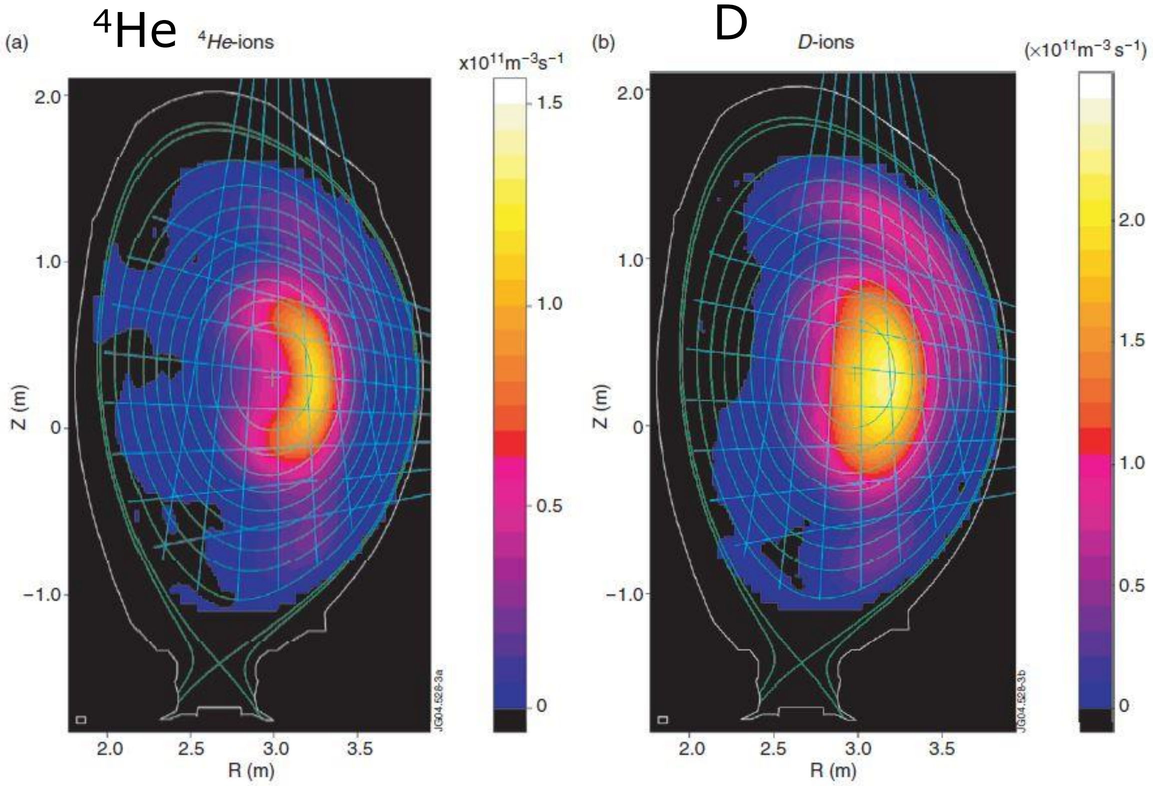
Prior 2: We think it ought to be fairly uniform and shouldn’t have large peaks, so penalize large values or large gradients.

Solution	$\sum x_i^2$	$\sum_{neighbors} (x_i - x_j)^2$	
0 3 / 4 3	32	26	
1 2 / 3 4	30	10	Most uniform, least spiky
2 1 / 2 5	34	26	
3 0 / 1 6	46	74	

γ-ray camera at JET

- γ-ray emission from D and ^{4}He accelerated by ICRF
- D and ^{4}He have the same charge-to-mass ratio *q/m*
- Typically fast-ion energy >0.5 MeV for significant γ-ray production

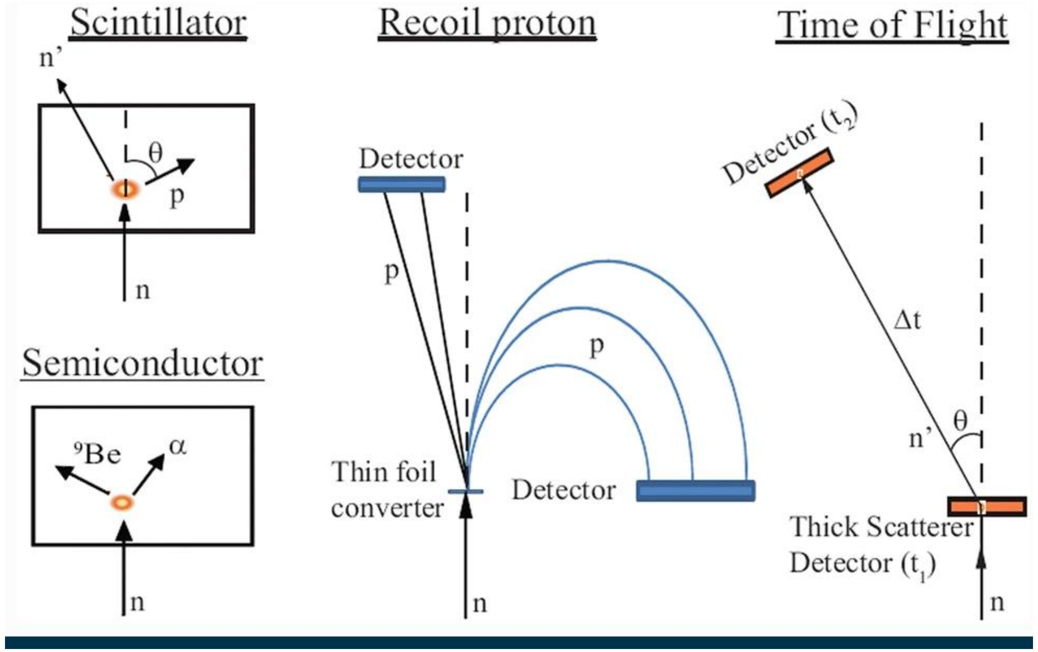


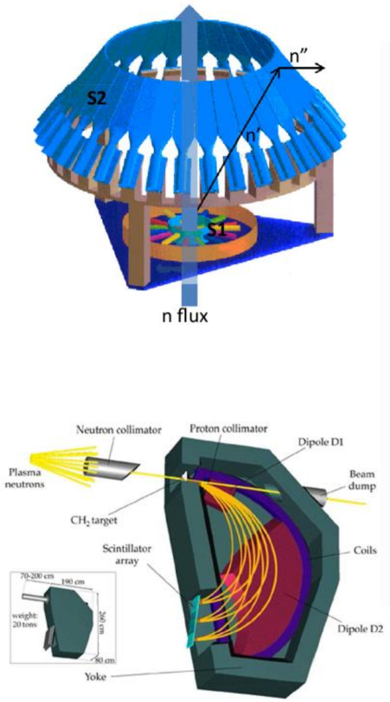


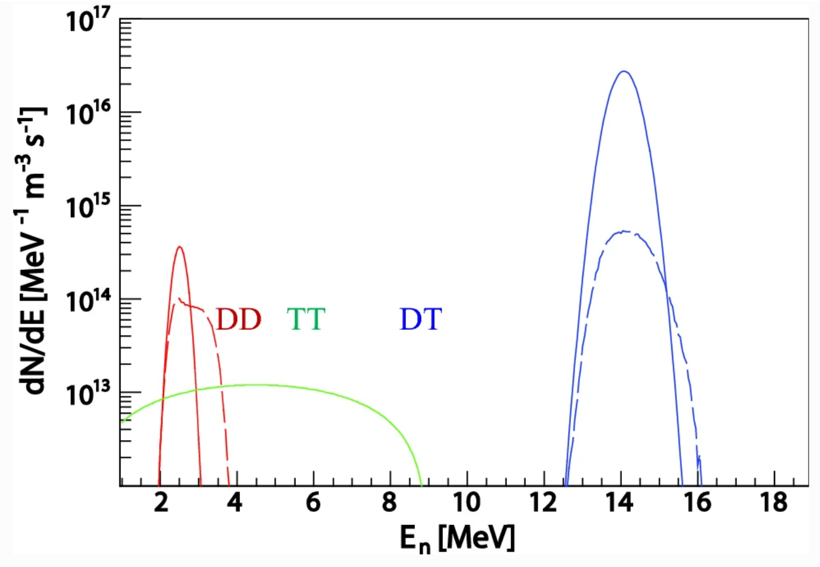
Kiptily et al. (2005) NF

Typical neutron emission spectrometry diagnostics (NES)

- Time-of-flight detector TOFOR – measure time of flight between 2 detections
- Magnetic proton recoil detector – measure radius of protons generated by neutrons
- Diamond semiconductor detector – measure energy deposited in a single crystal diamond
- Liquid scintillator detector – meausure light generated by neutron impact on scintillator







Ericsson et al. (2019) J. Fusion Energy

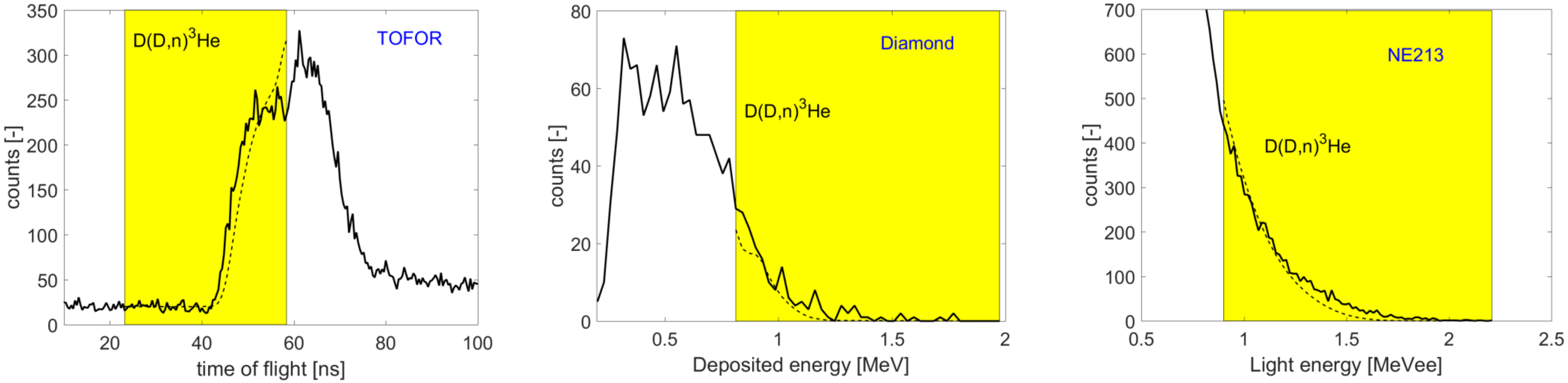
Neutron emission spectrometry measurements (NES)

- 3 simultaneously measured neutron emission spectra in JET #86459
- 4.5 MW NBI + 3 MW 3rd harmonic ICRF heating
- Yellow: Sensitive to only ICRF ions, but not NBI ions
- Measurement units:

  -TOFOR: time of flight

  -Diamond: energy deposited in a single crystal diamond

  -NE213 scintillator: energy of an equivalent electron that would have caused the same light flash



Eriksson et al (2015) NF, Salewski et al. (2017) NF

Neutron emission spectroscopy (NES) – spectrum formation

$D + D \rightarrow {}^3He\ (0.82\ MeV) + n\ (2.45\ MeV)$

$D + T \rightarrow {}^4He\ (3.5\ MeV) + n\ (14.1\ MeV)$

- Energy and momentum conservation for a beam-target reaction, $v_f \gg v_r$

$$\frac{1}{2}m_f v_f^2 + \frac{1}{2}m_r v_r^2 + Q = \frac{1}{2}m_{pr} v_{pr}^2 + \frac{1}{2}m_n v_n^2$$

$$m_f \boldsymbol{v}_f + m_r \boldsymbol{v}_r = m_{pr} \boldsymbol{v}_{pr} + m_n \boldsymbol{v}_n$$

- To eliminate v_{pr} in the energy equation, solve the momentum equation for v_{pr} and square:

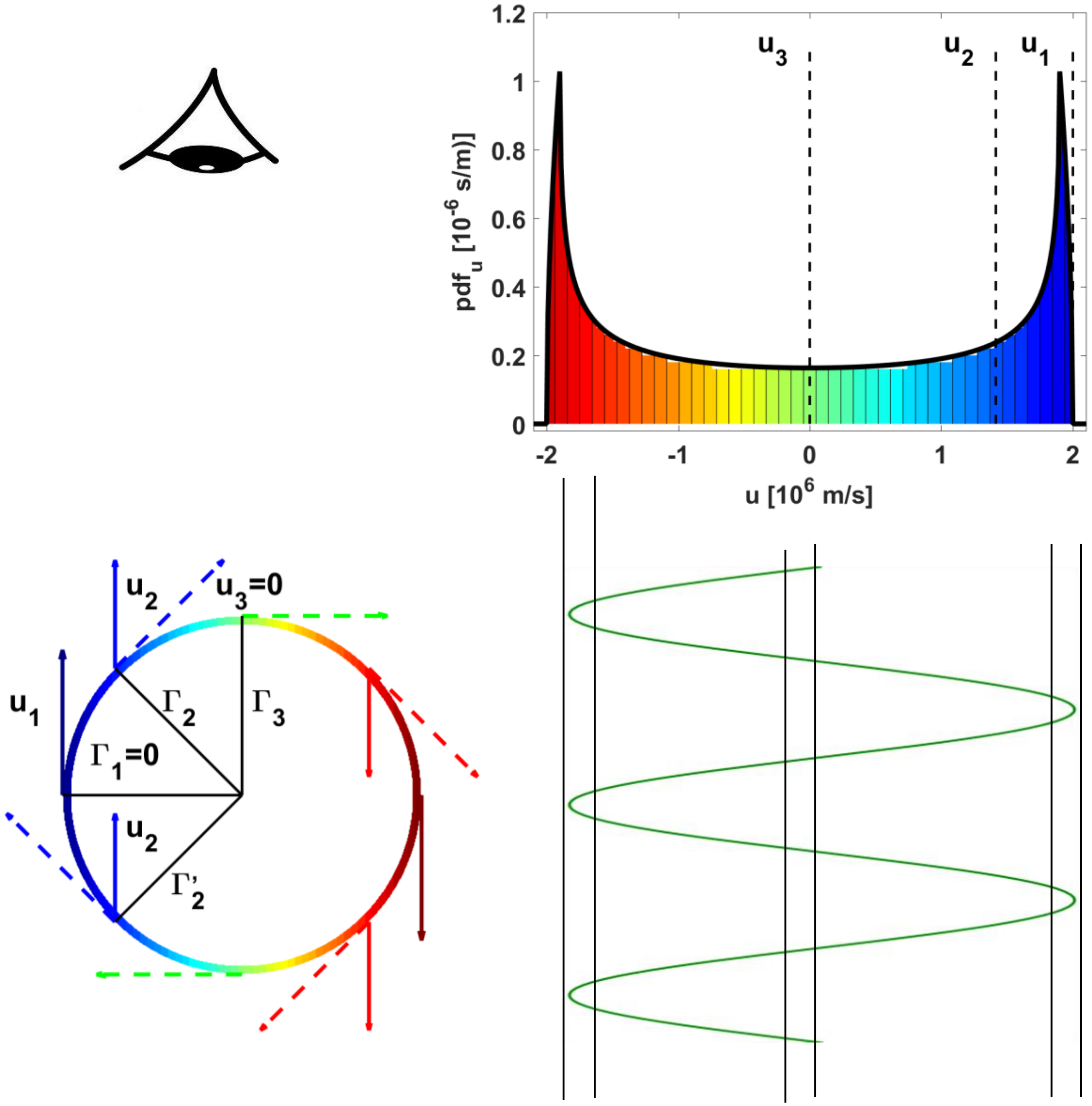
$$v_{pr}^2 = \frac{1}{m_{pr}^2}\left(m_f \boldsymbol{v}_f - m_n \boldsymbol{v}_n\right)^2 = \frac{1}{m_{pr}^2}\left(m_f^2 v_f^2 - 2 m_f m_n \boldsymbol{v}_f \cdot \boldsymbol{v}_n + m_n^2 v_n^2\right)$$

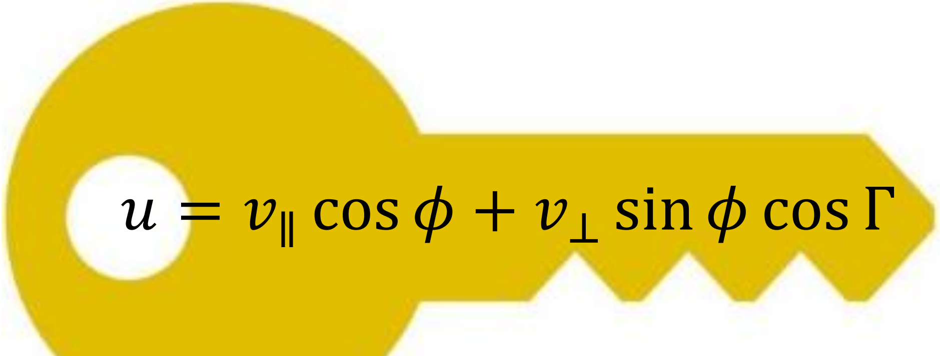
- Crucial term: projection of the ion velocity on the neutron velocity:

$$\boldsymbol{v}_f \cdot \boldsymbol{v}_n = u v_n$$

- u: projected velocity for the ion onto the line-of-sight of the detector
- v_n: neutron speed, which we get from the measurement

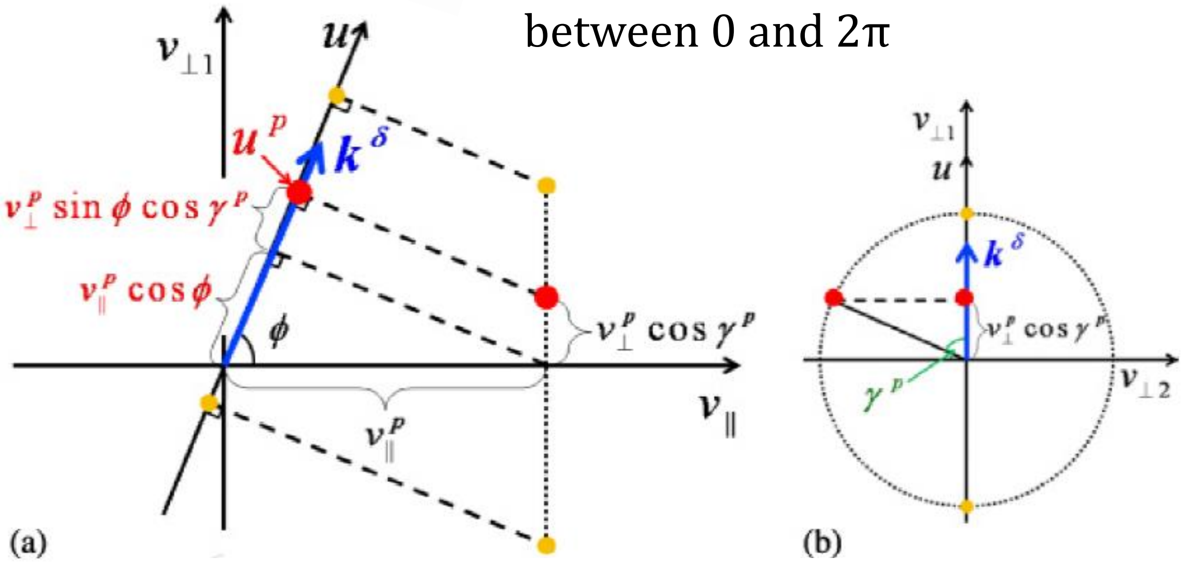
The projected velocity *u* onto the line-of-sight – the key to phase-space





$$u = v_\parallel \cos\phi + v_\perp \sin\phi \cos\Gamma$$

- ϕ: angle between line-of-sight and ***B***
- Γ: gyroangle, varies uniformly between 0 and 2π



Salewski et al. 2011 NF, 2014 PPCF

Neutron emission spectroscopy (NES) – spectrum formation

- Eliminate v_{pr} and solve for the neutron energy E_n:

$$E_n = \frac{m_{pr}}{m_{pr}+m_n}Q + \frac{m_{pr}-m_f}{m_{pr}+m_n}\frac{1}{2}m_f v_f^2 + \frac{m_f m_n}{m_{pr}+m_n}u v_n$$

- 3 terms in the neutron energy E_n:
  - Part of reaction energy
  - Part of fast ion energy
  - Depends on projected velocity u onto line-of-sight

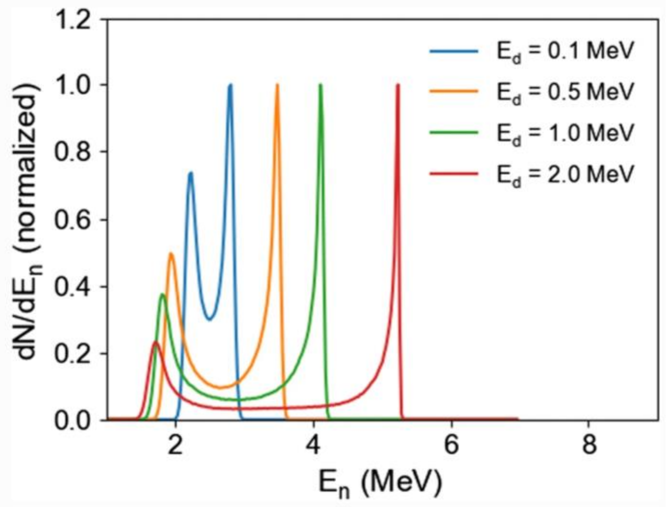
$$u = v_\parallel \cos\phi + v_\perp \sin\phi \cos\Gamma$$

- For beam-target reactions:

$$E_n = \frac{m_{pr}}{m_{pr}+m_n}Q + \frac{m_{pr}-m_f}{m_{pr}+m_n}\frac{1}{2}m_f\left(v_\parallel^2 + v_\perp^2\right) + \frac{m_f m_n}{m_{pr}+m_n}(v_\parallel \cos\phi + v_\perp \sin\phi\cos\Gamma)v_n$$

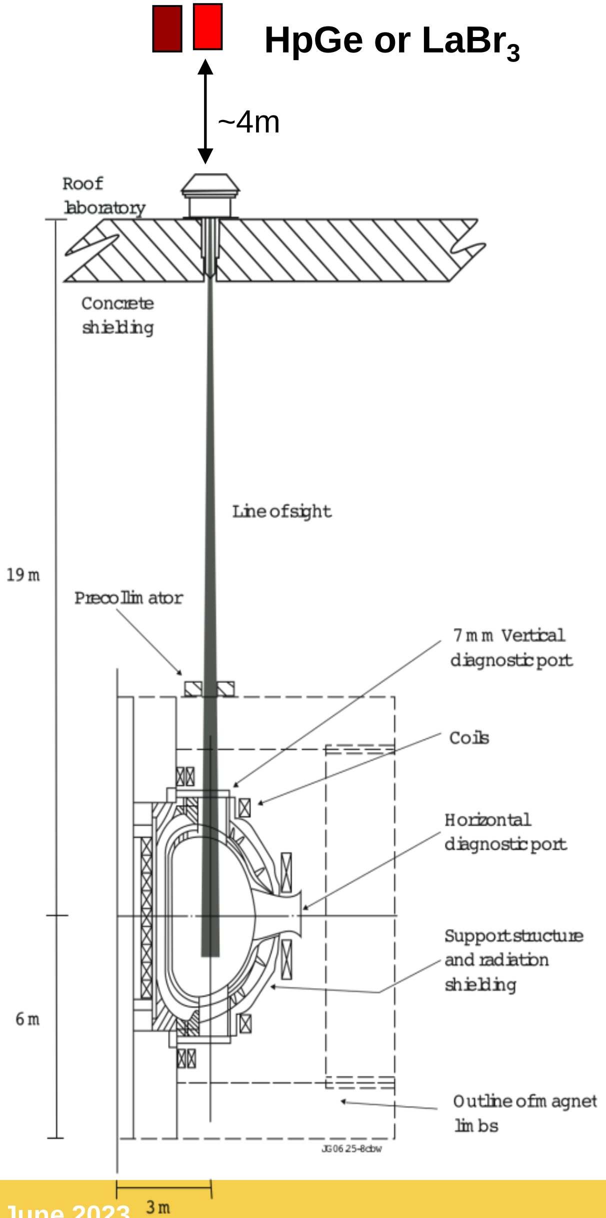
- General formula by Brysk (1975):

$$E_n = \frac{1}{2}m_n v_{cm}^2 + \frac{m_R}{m_n+m_R}(Q+K) + v_{cm}\cos(\theta)\left(\frac{2m_n m_R}{m_n+m_R}(Q+K)\right)^{1/2}$$



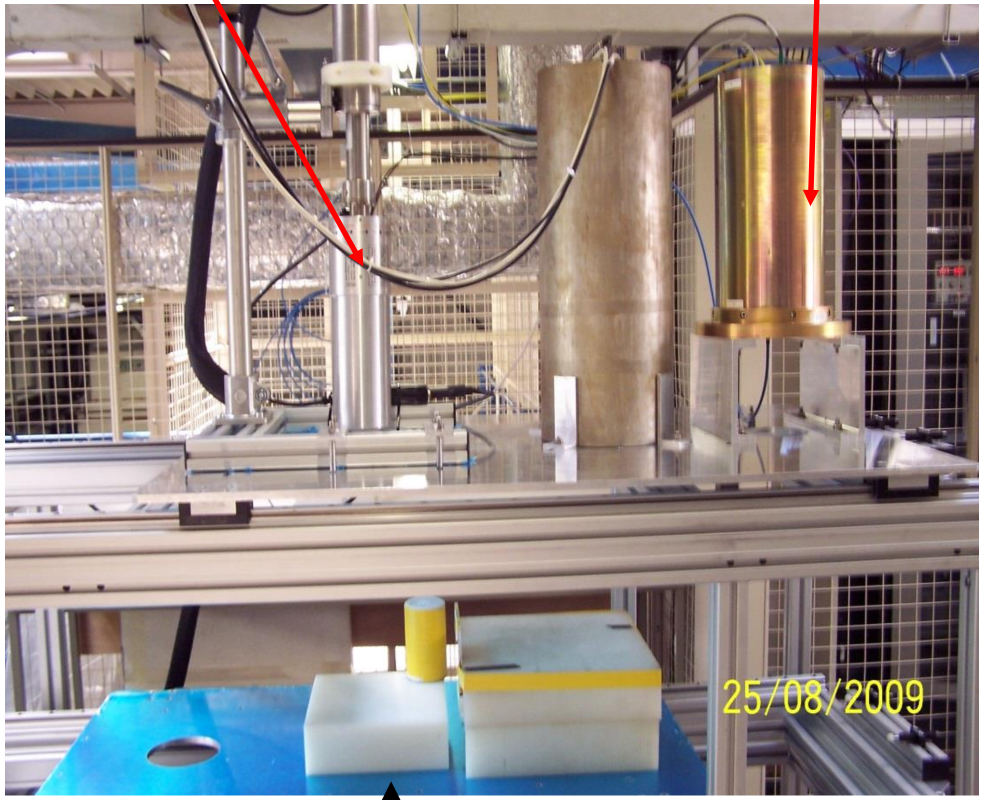
Ericsson et al. (2019) J. Fusion Energy

γ-ray spectroscopy (GRS) at JET



HpGe, very high energy resolution

$LaBr_3$, MHz rate at high energy resolution



γ-rays, neutrons

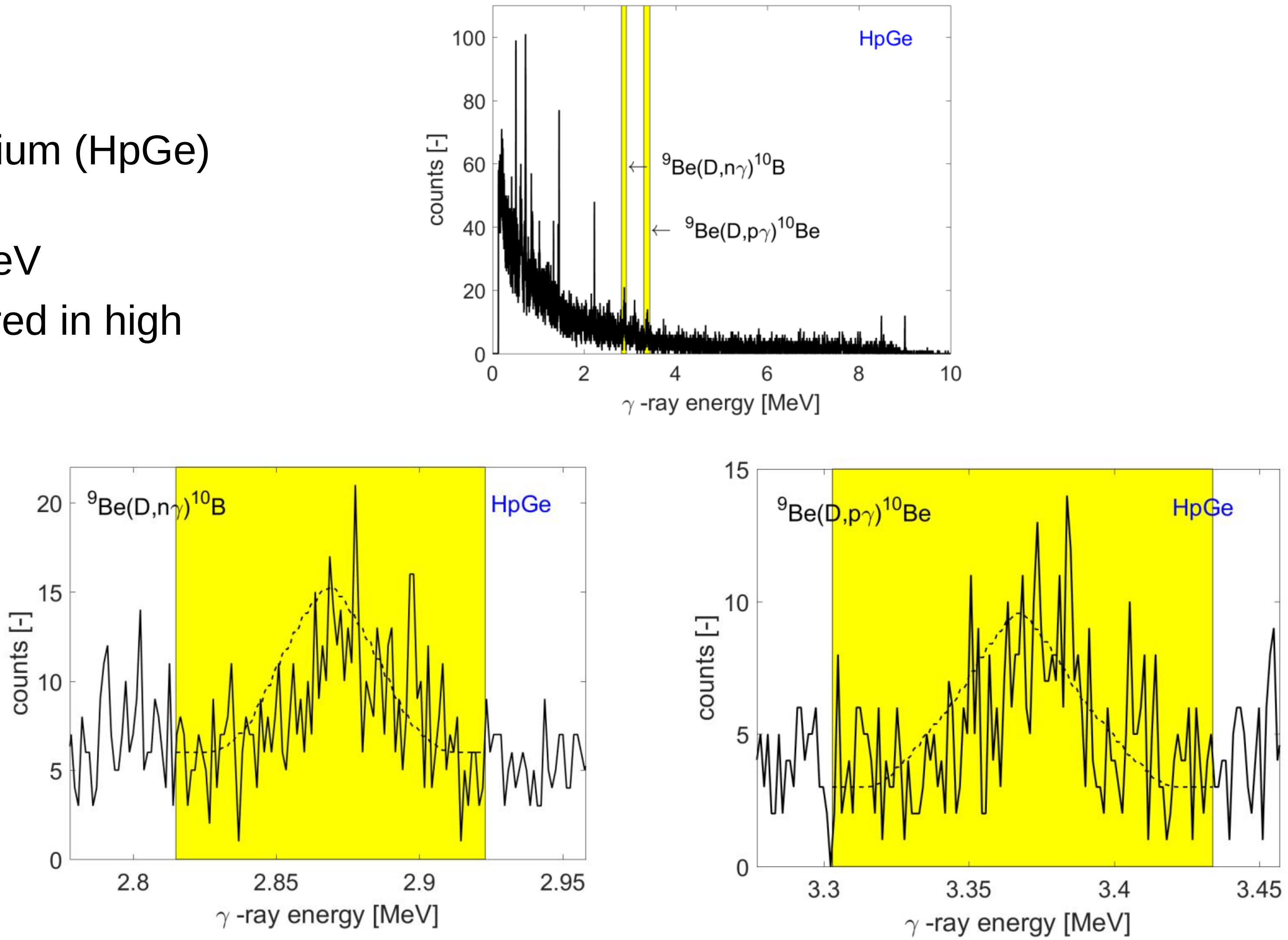
Nocente et al. (2010) RSI

γ-ray spectroscopy measurements (GRS)

- γ-ray spectra in JET #86459
- High-resolution High-purity Germanium (HpGe) detector
- Energy resolution: 1 keV over 10 MeV
- Many peaks simultaneously measured in high resolution
- Two example reactions:

$$D + {}^{9}Be \rightarrow n + {}^{10}B^{*}$$
$${}^{10}B^{*} \rightarrow {}^{10}B + \gamma\ (2.868\ MeV)$$

$$D + {}^{9}Be \rightarrow p + {}^{10}Be^{*}$$
$${}^{10}Be^{*} \rightarrow {}^{10}Be + \gamma\ (3.367\ MeV)$$



Eriksson et al (2015) NF, Salewski et al. (2017) NF

One-step reaction γ-ray spectroscopy (GRS) – spectrum formation

- Weak branches of DT reaction emit γ-rays: alternative diagnostic for the fusion yield $D + T \rightarrow {}^5He + \gamma\ (16.7\ MeV),\ D + T \rightarrow {}^5He + \gamma\ (\sim 13.5\ MeV)$

- Energy and momentum for beam-target reaction

$$\frac{1}{2} m_f v_f^2 + \cancel{\frac{1}{2} m_r v_r^2} + Q = \frac{1}{2} m_{pr} v_{pr}^2 + E_\gamma$$

$$m_f \boldsymbol{v}_f + \cancel{m_r \boldsymbol{v}_r} = m_{pr} \boldsymbol{v}_{pr} + \boldsymbol{p}_\gamma$$

- Same procedure as for neutrons: isolate $\boldsymbol{v}_{pr}$ in momentum, square, projected velocity appears

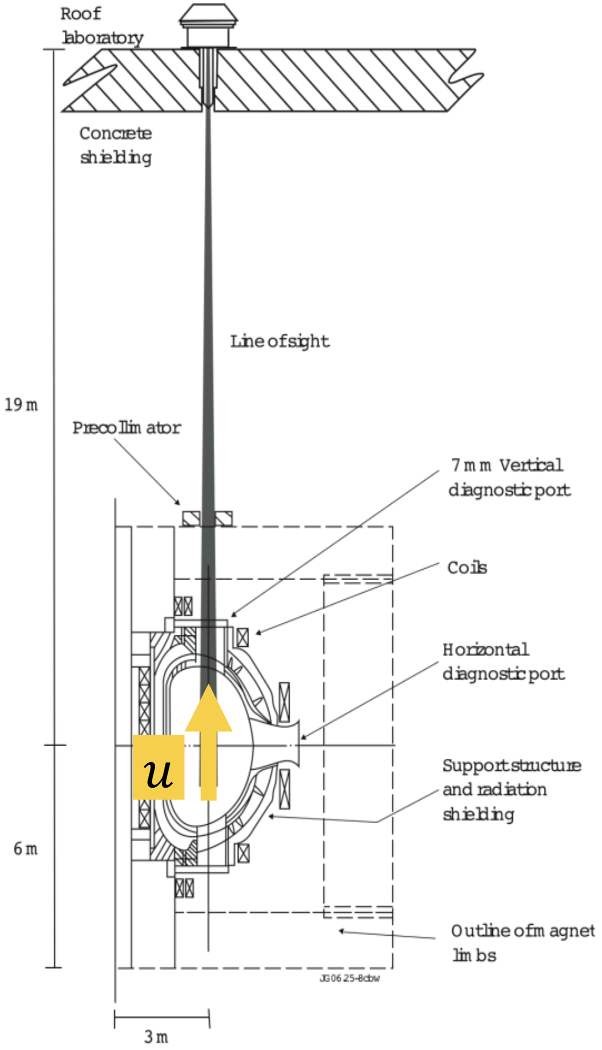
$$m_{pr}^2 v_{pr}^2 = m_f^2 v_f^2 + p_\gamma^2 - 2 m_f \boldsymbol{v}_f \cdot \boldsymbol{p}_\gamma = m_f^2\ v_f^2 + p_\gamma^2 - 2 m_f u p_\gamma$$

- Eliminate v_{pr}, solve for E_γ, Taylor expansion in *u/c*

$$E_\gamma = p_\gamma c$$

$$E_\gamma = \left(1 + \frac{m_f}{m_{pr}} \frac{u}{c}\right)\left(Q + \left(1 - \frac{m_f}{m_{pr}}\right) E_f\right)$$

$$u = v_\parallel \cos\phi + v_\perp \sin\phi \cos\Gamma$$



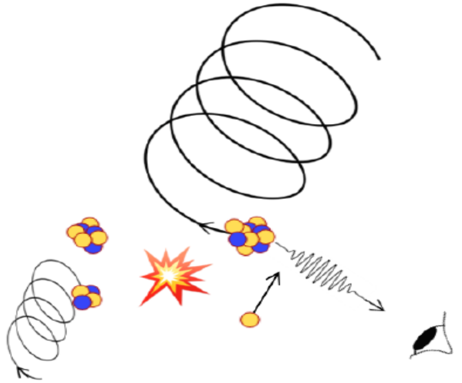
Two-step reaction γ-ray spectroscopy (GRS) – spectrum formation

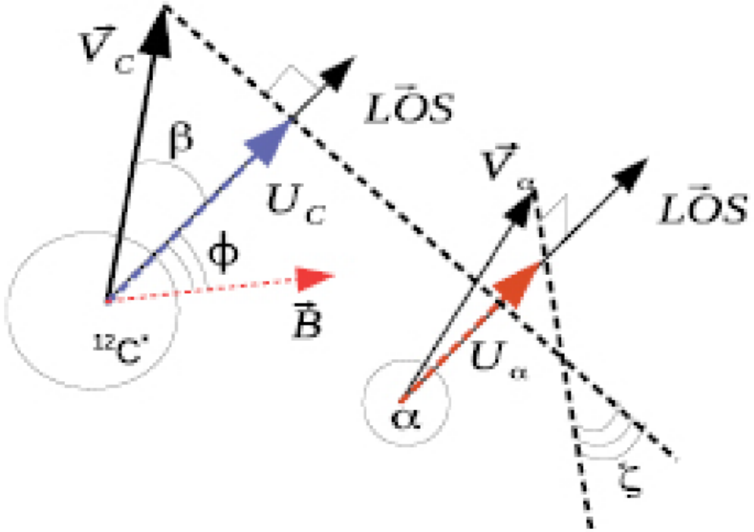
- Same procedure as for neutron emission and one-step γ-ray spectroscopy, but for both steps of the two-step reaction
- Step 1: Energy and momentum conservation for beam-target reaction, solve for the velocity of the excited species
- Step 2: Energy and momentum conservation for the de-excitation (Doppler shift, Fermi (1932) Rev. Mod. Phys.)

$$E_\gamma = E_{\gamma 0}\left(1 + \frac{u_{pr}}{c}\right)$$

$$u_{\mathrm{pr}} = \frac{m_{\mathrm{f}}}{m_{\mathrm{pr}} + m_{\mathrm{n}}}\cos\beta\left(u\cos\beta + \sqrt{v_{\mathrm{f}}^2 - u^2}\sin\beta\cos\zeta\right) \pm \sqrt{\cos^2\beta\left(\frac{m_{\mathrm{f}}^2}{(m_{\mathrm{pr}}+m_{\mathrm{n}})^2}\left(u\cos\beta + \sin\beta\cos\zeta\sqrt{v_{\mathrm{f}}^2 - u^2}\right)^2 + \frac{2m_{\mathrm{n}}}{m_{\mathrm{pr}}(m_{\mathrm{pr}}+m_{\mathrm{n}})}Q^* - \frac{m_{\mathrm{f}}(m_{\mathrm{f}}-m_{\mathrm{n}})}{m_{\mathrm{pr}}(m_{\mathrm{pr}}+m_{\mathrm{n}})}\right.}$$

$$u = v_\parallel \cos\phi + v_\perp \sin\phi \cos\Gamma$$





Salewski et al. (2015) NF

Can we tell energetic α-particles apart from energetic deuterium or tritium in neutron emission spectroscopy (NES) and γ-ray spectroscopy (GRS) measurements?

1) NES yes, GRS yes
2) NES yes, GRS no
3) NES no, GRS yes
4) NES no, GRS no
5) I am not sure.

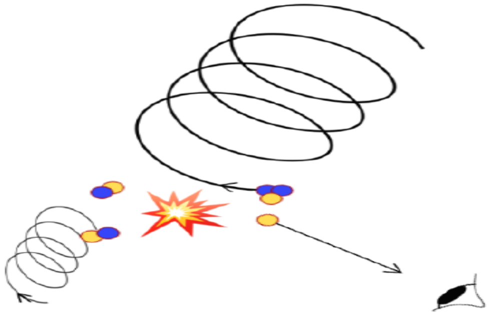
$$D + {}^{9}Be \rightarrow n + {}^{10}B^{*}$$
$${}^{10}B^{*} \rightarrow {}^{10}B + \gamma\ (2.868\ MeV)$$

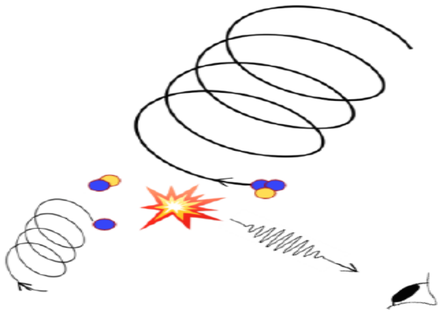
$$D + {}^{9}Be \rightarrow p + {}^{10}Be^{*}$$
$${}^{10}Be^{*} \rightarrow {}^{10}Be + \gamma\ (3.367\ MeV)$$

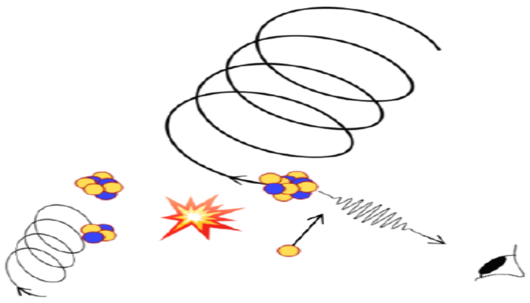
$$\alpha + {}^{9}Be \rightarrow n + {}^{12}C^{*(1,2)}$$
$${}^{12}C^{*(1)} \rightarrow {}^{12}C + \gamma\ (4.44\ MeV)$$

$$D + T \rightarrow \alpha\ (3.5\ MeV) + n\ (14.1\ MeV)$$

$$D + T \rightarrow {}^{5}He + \gamma\ (16.7\ MeV)$$







Neutron emission spectroscopy (NES)

γ-ray spectroscopy (GRS)

Can we tell energetic α-particles apart from energetic deuterium or tritium in neutron emission spectroscopy (NES) and γ-ray spectroscopy (GRS) measurements?

1) NES yes, GRS yes
2) NES yes, GRS no
3) NES no, GRS yes
4) NES no, GRS no
5) I am not sure.

We can tell the reaction from the detected energies.

$$D + {}^{9}Be \rightarrow n + {}^{10}B^{*}$$
$${}^{10}B^{*} \rightarrow {}^{10}B + \gamma\ (2.868\ MeV)$$

$$D + {}^{9}Be \rightarrow p + {}^{10}Be^{*}$$
$${}^{10}Be^{*} \rightarrow {}^{10}Be + \gamma\ (3.367\ MeV)$$

$$\alpha + {}^{9}Be \rightarrow n + {}^{12}C^{*(1,2)}$$
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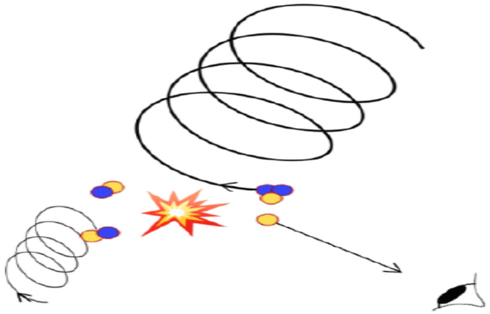
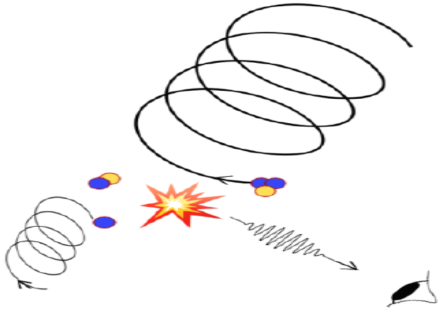
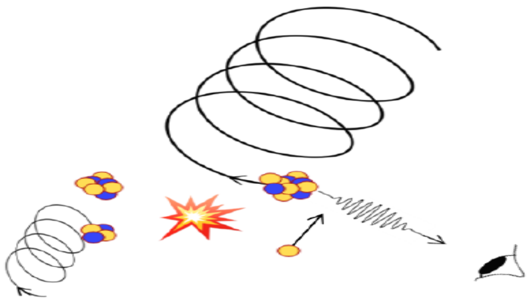
$$D + T \rightarrow {}^{5}He + \gamma\ (16.7\ MeV)$$



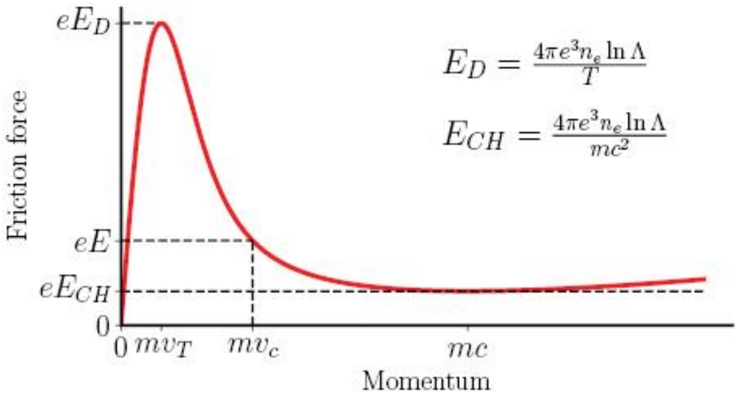




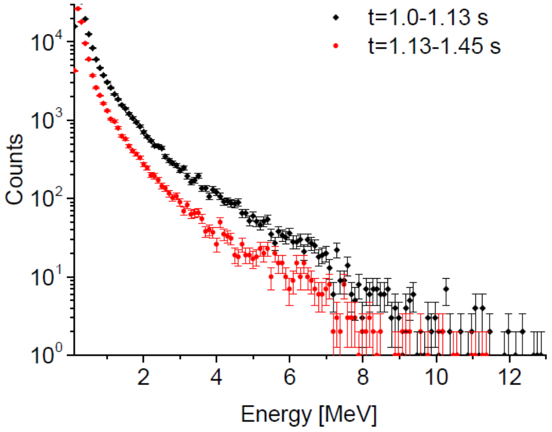
Neutron emission spectroscopy (NES)

γ-ray spectroscopy (GRS)

Measurements of runaway electrons by γ-ray spectroscopy (GRS)



- Measured bremsstrahlung



Nocente et al. (2018) RSI

Breizman et al. (2019) NF

- Parallel electric fields can accelerate electrons to high energies.
- The faster an energetic electron, the lower the Coulomb friction force, leading to electron runaway.
- Classically, the radiated power of an accelerated charge is

$$P = \frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}\right)^2 \qquad \text{(CGS units)}$$

- For relativistic electrons in a spatially uniform field, the radiated power due to the gyro-motion is

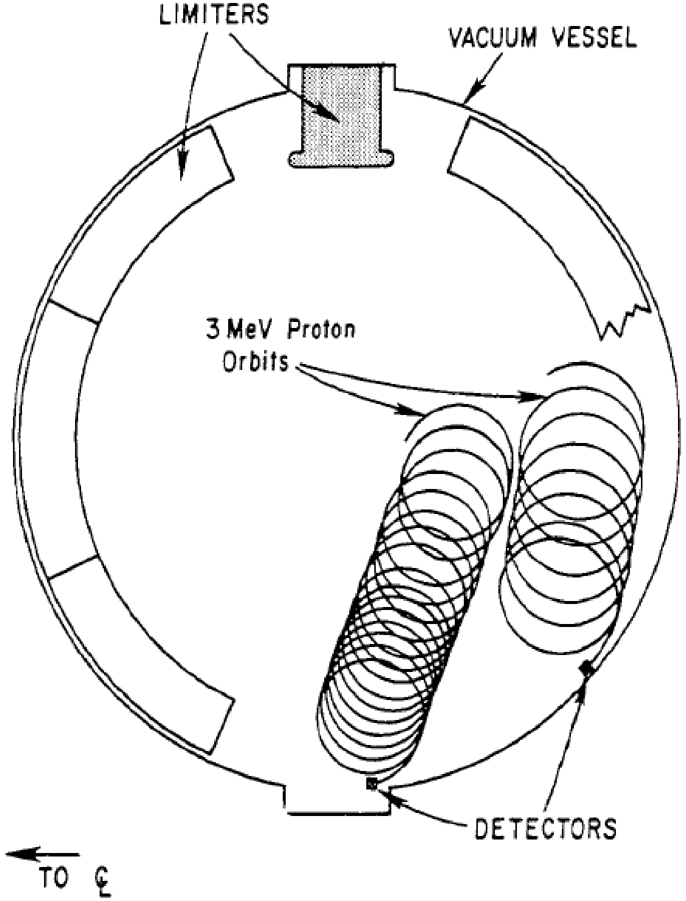
$$P = \frac{2e^4B^2}{3m^4c^7}p^2c^2\sin^2\theta \qquad \text{(CGS units)}$$

- θ is the pitch angle of the runaway electron
- In addition to gyration, bremsstrahlung is emitted due to collisions. The bremsstrahlung is in the MeV range, which is detectable by γ-ray spectrometers.

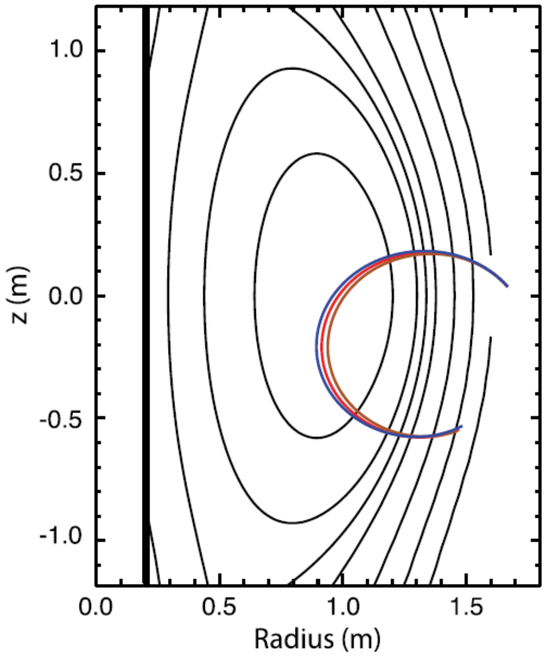
See lectures by R. Granetz and T. Fülöp on Friday

Charged fusion product spectroscopy – 3 MeV protons

3 MeV proton diagnostic
Princeton Large Torus



3 MeV proton
diagnostic for
MAST



- Reaction kinematics similar to neutron emission spectrometry
- “Sightlines” are curved
- DD reaction produces 3 MeV protons

$$D + D \rightarrow p\,(3\ MeV) + T$$

$$D + {}^{3}He \rightarrow p\,(15\ MeV) + {}^{4}He$$

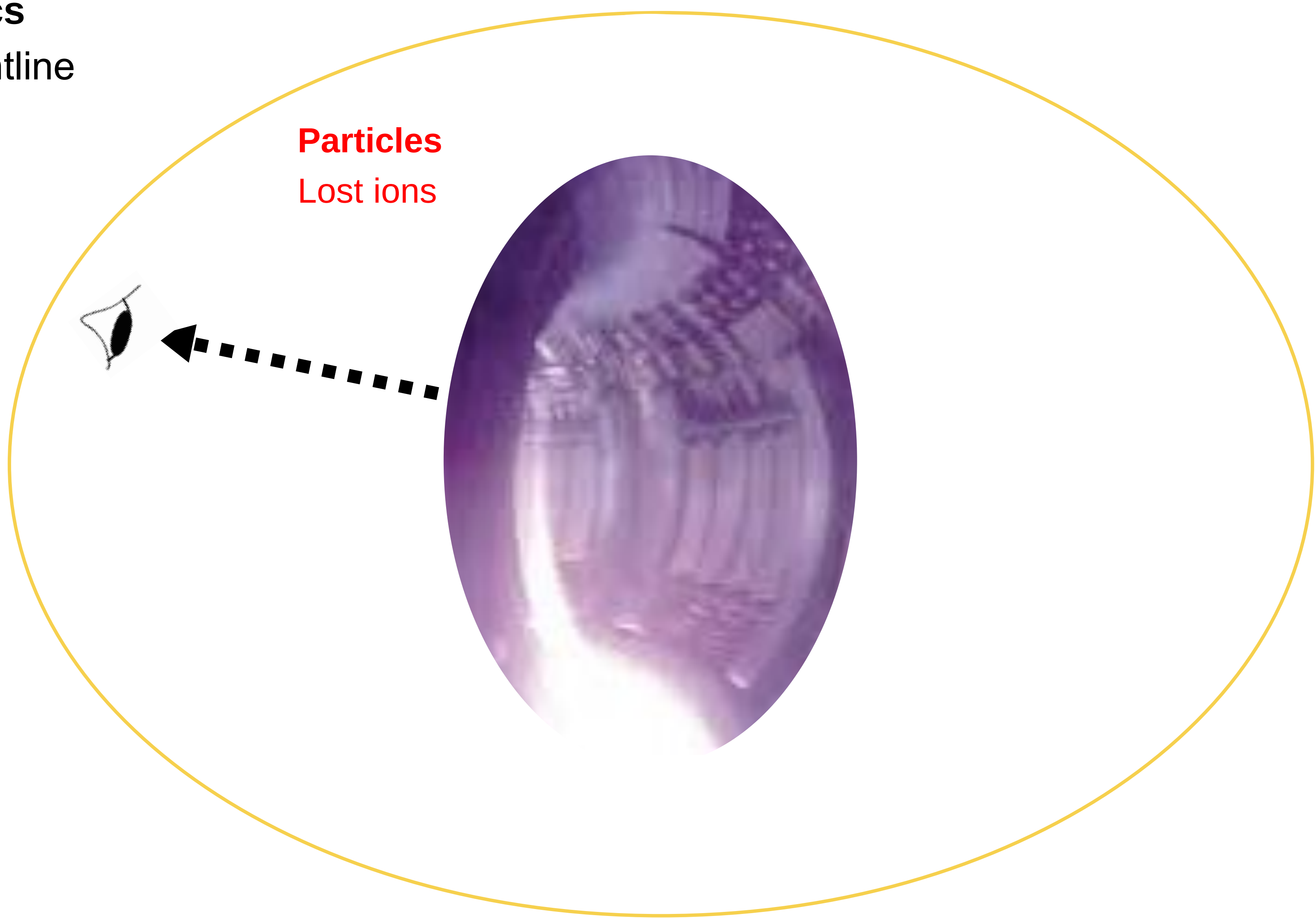
Heidbrink et al. (1986) PPCF, Heidbrink et al. (2021) NF

Energetic particle diagnostics

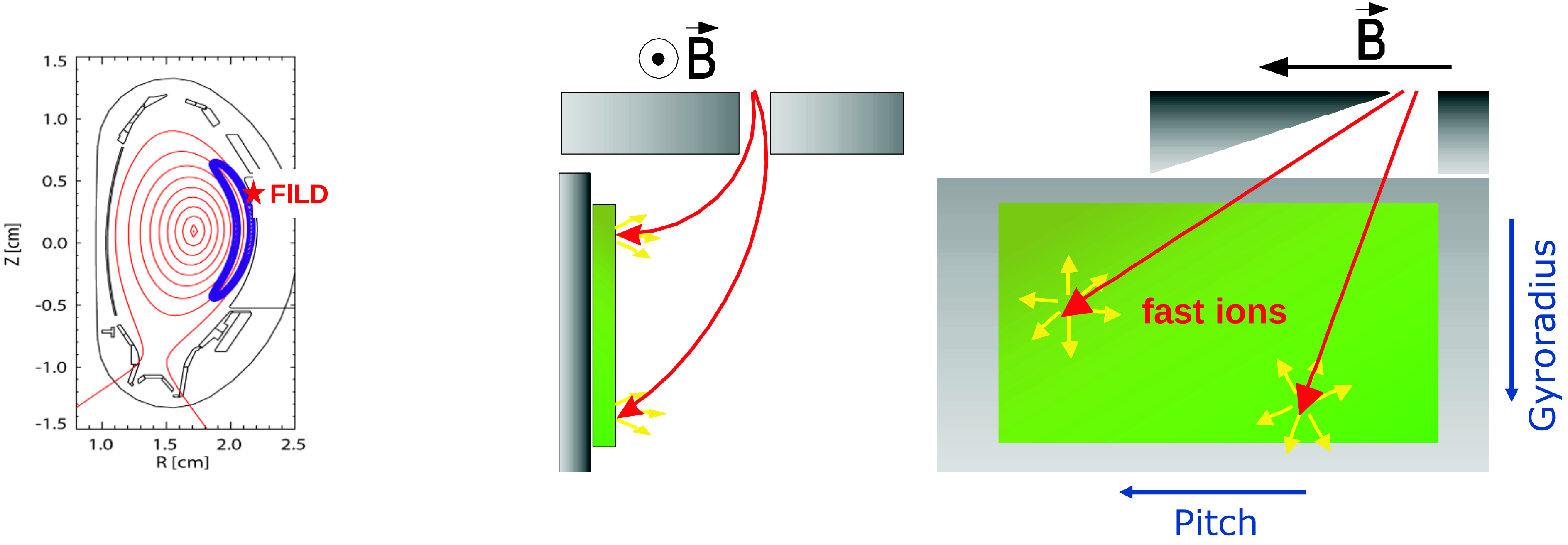
Passive diagnostics

•Typically along sightline

•Fast-ion loss detector



Fast-ion loss detector (FILD)



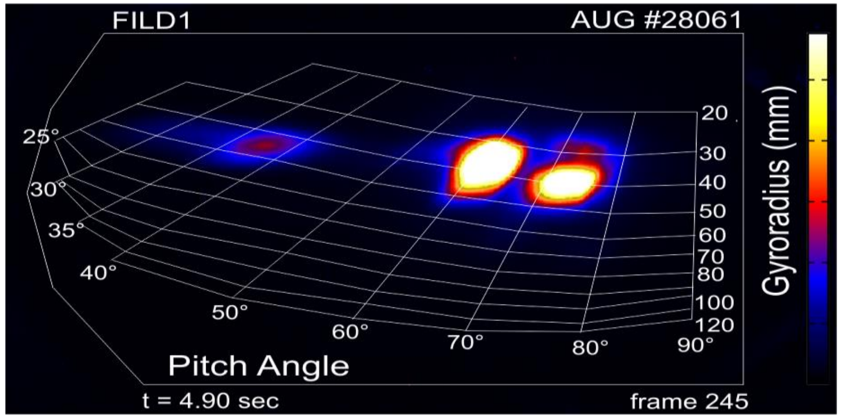
- Fast ion loss detectors measure ions lost from the plasma.
- Lost ions hit a scintillator causing a light flash that is photographed or send in a photomultiplier.
- Other designs use Faraday cups, measure currents in metal foils absorbing the ion.

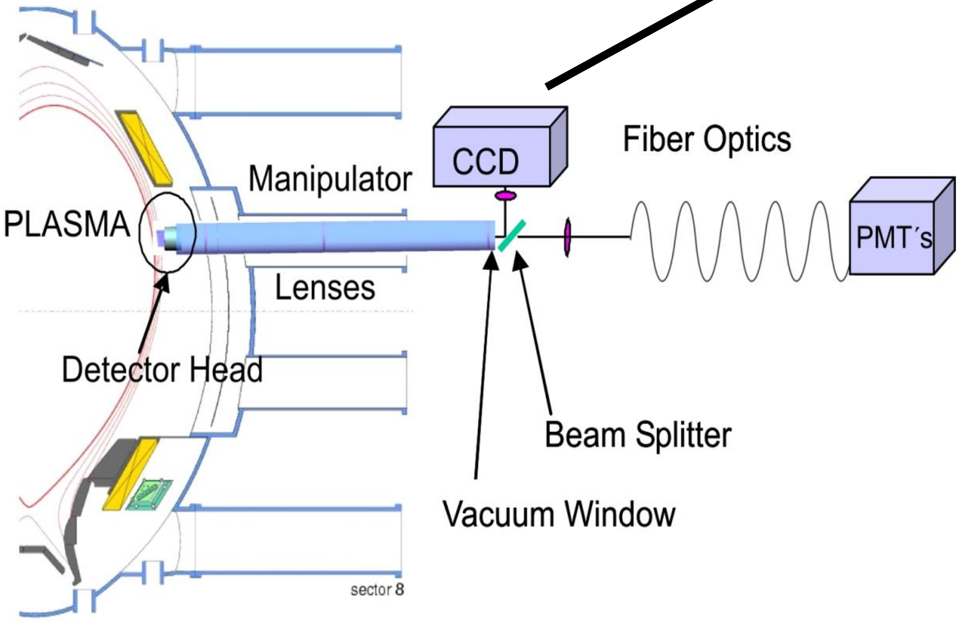
Zweben et al. (1988) NF, Garcia-Munoz et al. (2009) RSI

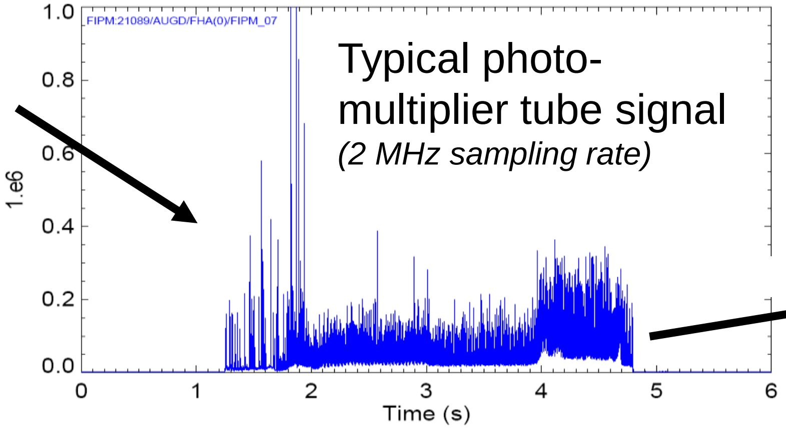
Fast-ion loss detector (FILD) measurements

- CCD camera (high spatial resolution)
- Photomultiplier tubes (MHz - temporal resolution)

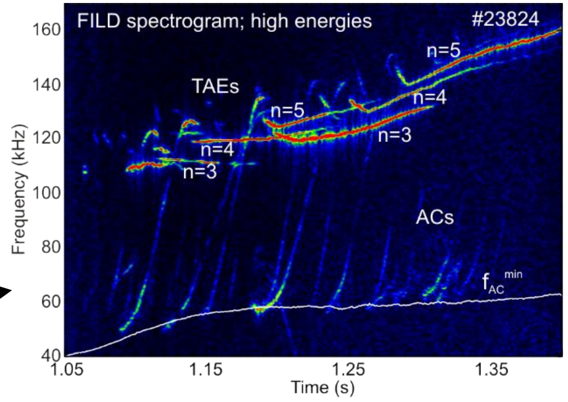
Typical CCD scintillator image







Typical spectrogram



Garcia-Munoz et al, (2010) PRL

See lecture by M. Garcia-Munoz

Energetic particle diagnostics: Neutral particle analyzers

Passive diagnostics

•Typically along sightline

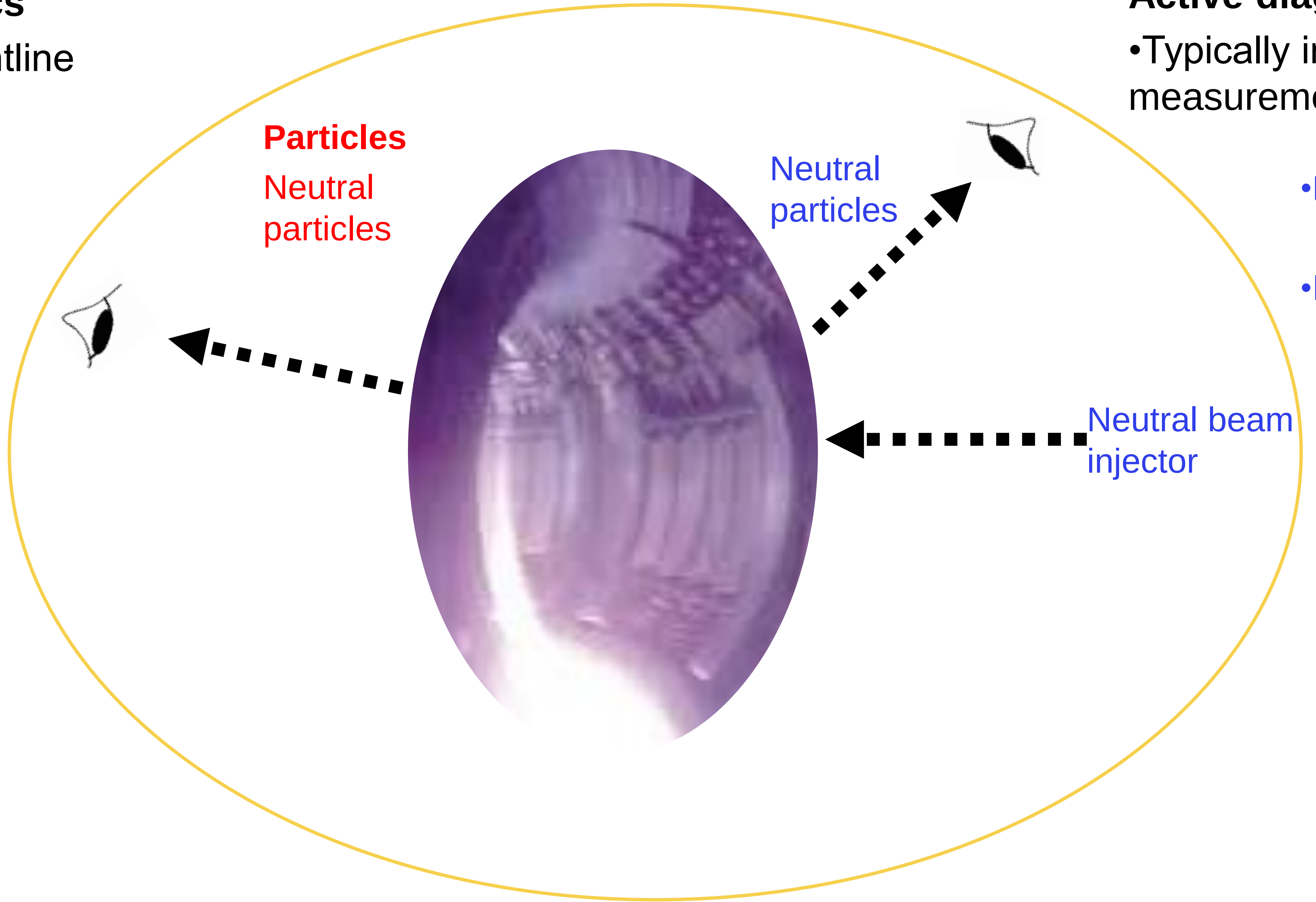
•Passive neutral particle analyser

Active diagnostics

•Typically in a small measurement volume

•Neutral particle analyser

•Imaging neutral particle analyzer



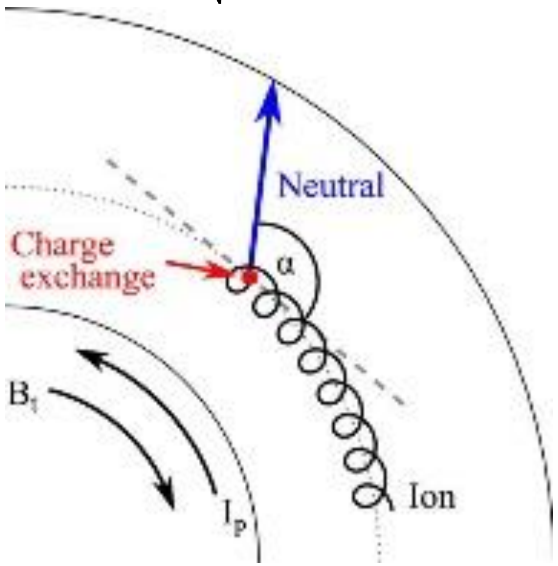
Neutral particle analyzers (NPA)

- Fast ions on helical trajectories
- Charge-exchange reaction with a neutral particle
- Fast ion is then neutral and proceeds along a straight line
- Ions with particular small ranges in gyro-angle and pitch reach the detector
- Detector measures the energy spectrum of neutral particles

- **Active**: Neutrals from neutral beam
- **Passive**: Neutrals from elsewhere, plasma edge

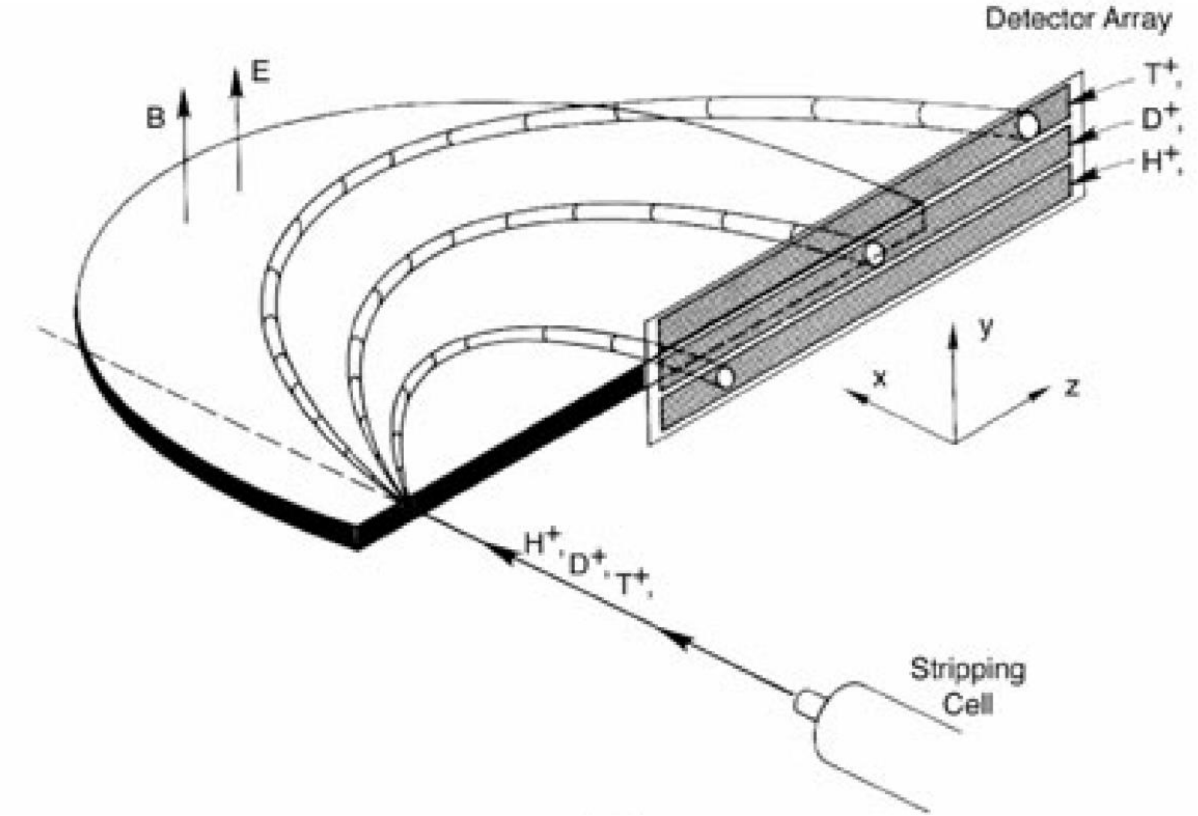
- To reach the detector from the plasma, the projected velocity and the velocity of the ion must be the same.

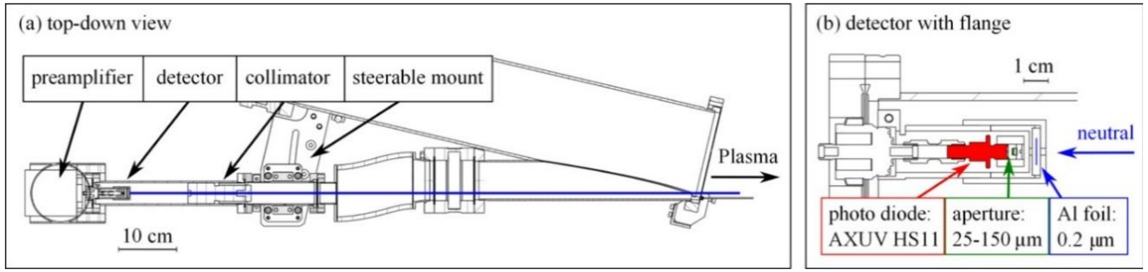
$$u = v = \sqrt{\frac{2E}{m}}$$



Schneider et al. 2015 RSI

Neutral particle analyzers (NPA)





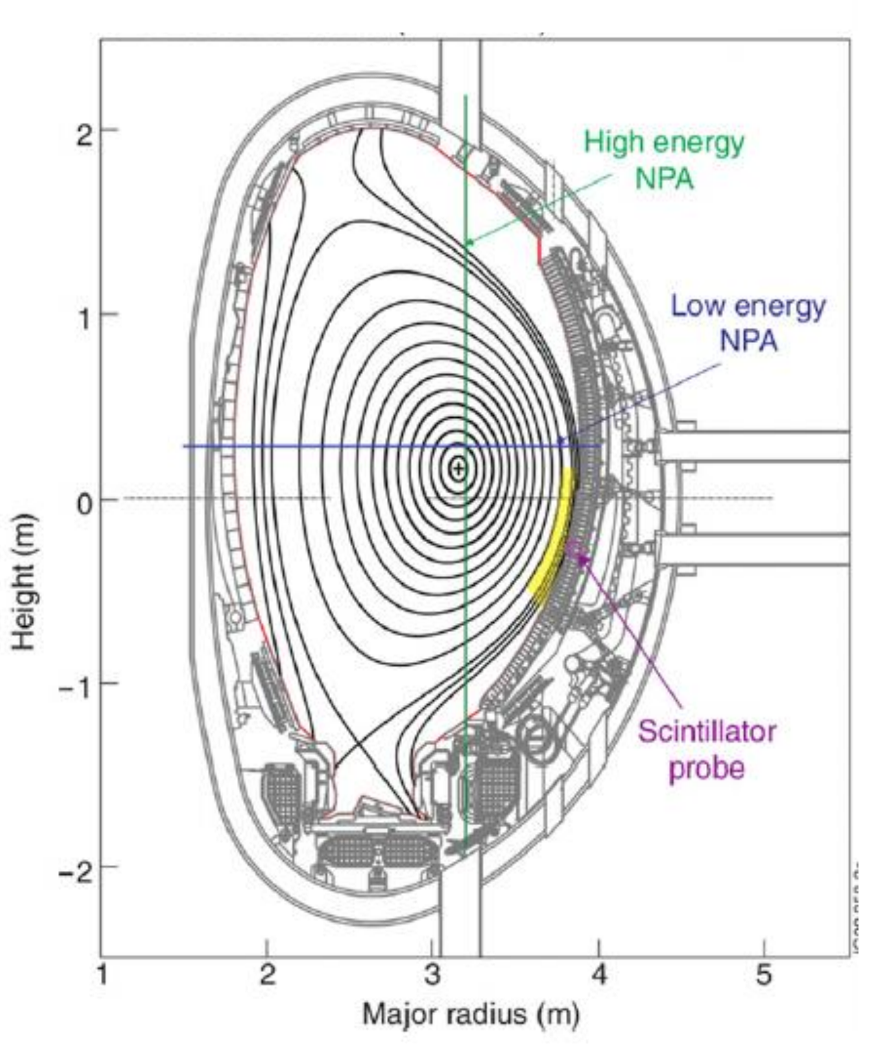
- Conventional neutral particle analyser, as in TFTR: E||B fields separate species in q/m and energy

- Solid state neutral particle analyser: charge pulse Q ~ energy

Kislyakov et al. 2008 FST

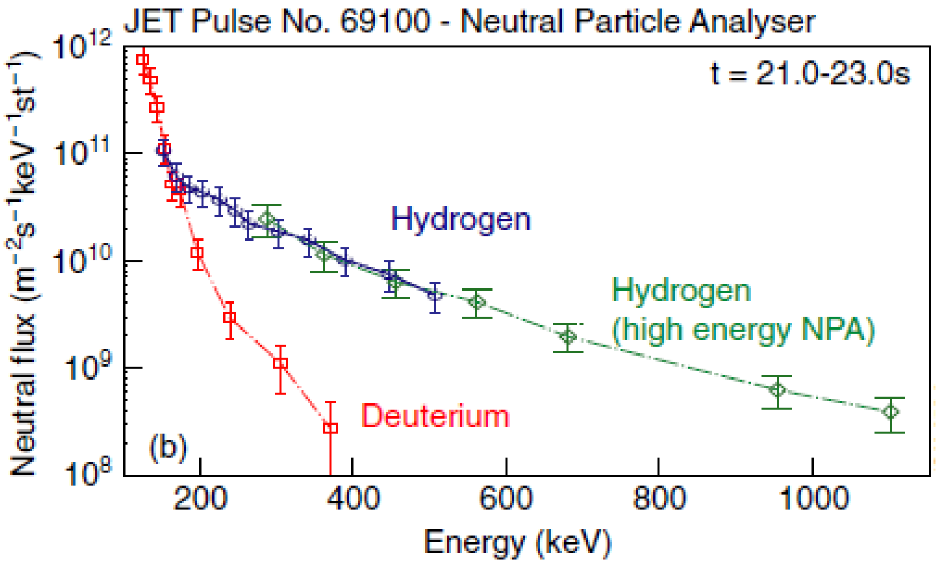
Schneider et al. 2015 RSI

Neutral particle analyzer (NPA) at JET

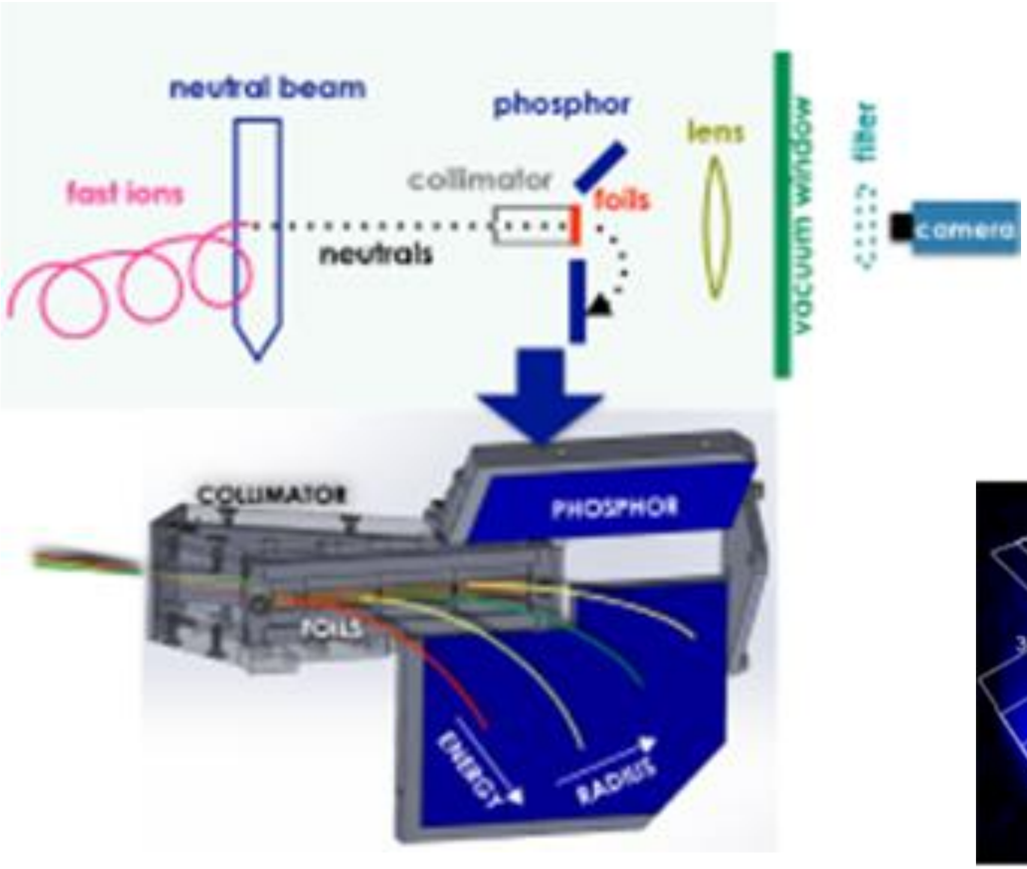


Perez von Thun et al. (2010) NF

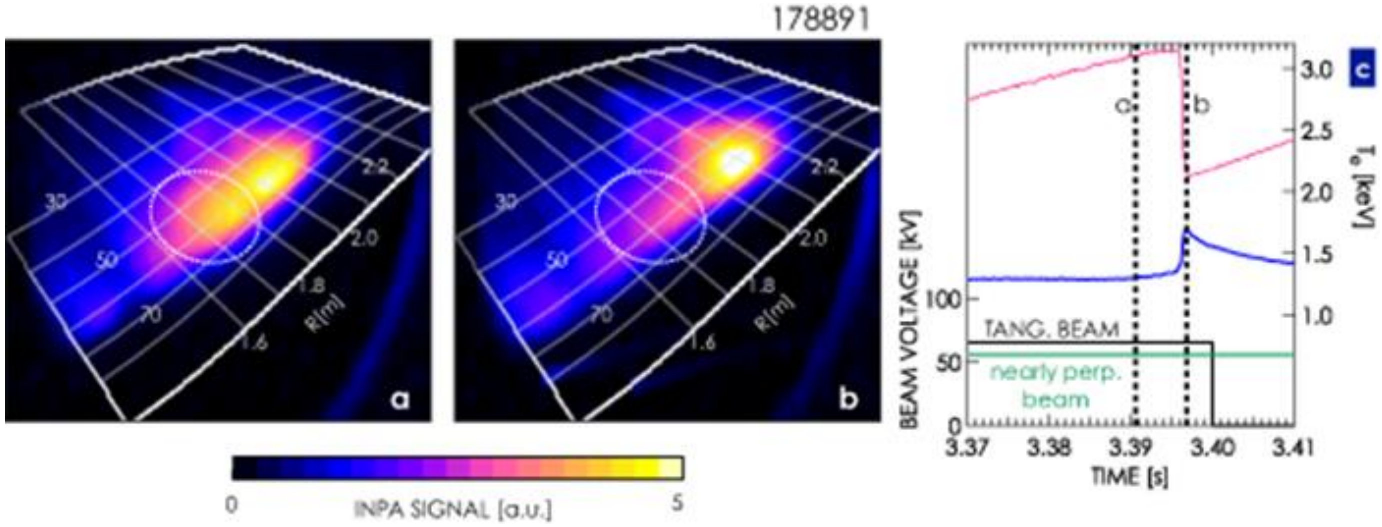
- High-energy and low-energy neutral particle analyzer at JET
- 15 MW NBI (E_0 = 130 keV in D), 6 MW ICRF heating (minority H)
- Energies above 130 keV are due to ICRF acceleration



Imaging neutral particle analyser (INPA) at DIII-D



- Measure a distribution in radius, energy (*R*,*E*) at a given pitch
- 1000s of simultaneous neutral particle analyzers pointed to different positions in major radius direction along the NBI
- Before and after a sawtooth crash
- Images show radial transport outward due to the sawtooth crash

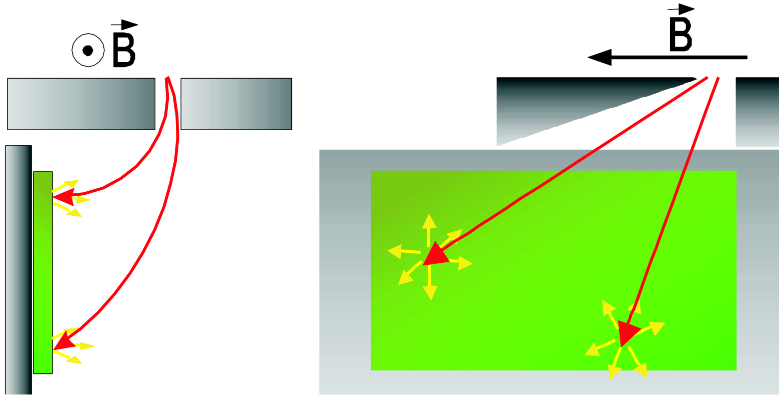


See lecture by M.A. van Zeeland

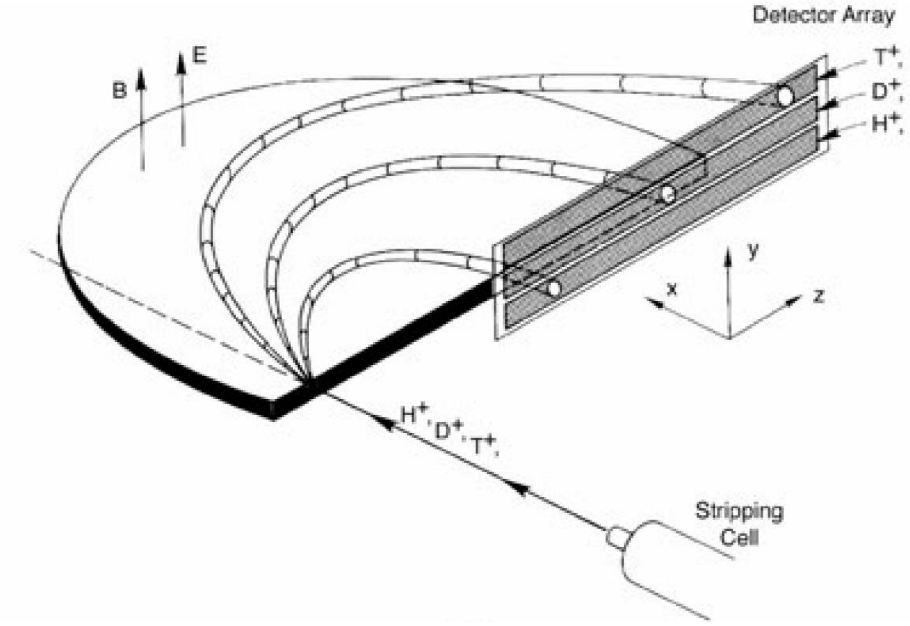
Du et al 2020 NF

Can we tell energetic α-particles and energetic deuterium apart in fast-ion loss detector (FILD) and neutral particle analyzer (NPA) measurements?

1) FILD yes, NPA yes
2) FILD yes, NPA no
3) FILD no, NPA yes
4) FILD no, NPA no
5) I am not sure.



Fast-ion loss detector (FILD)

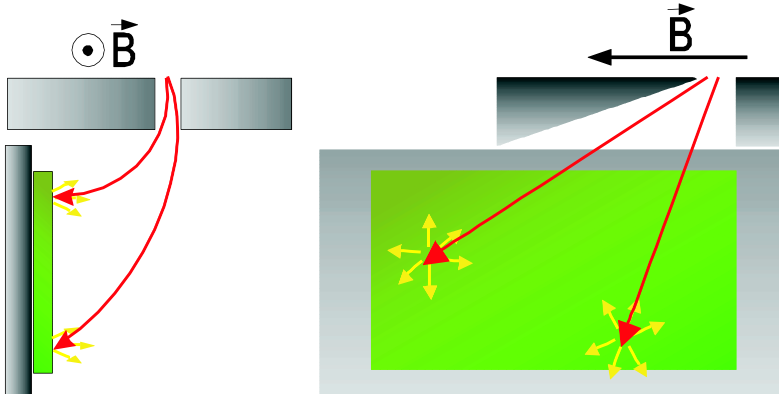


Neutral particle analyser (NPA)

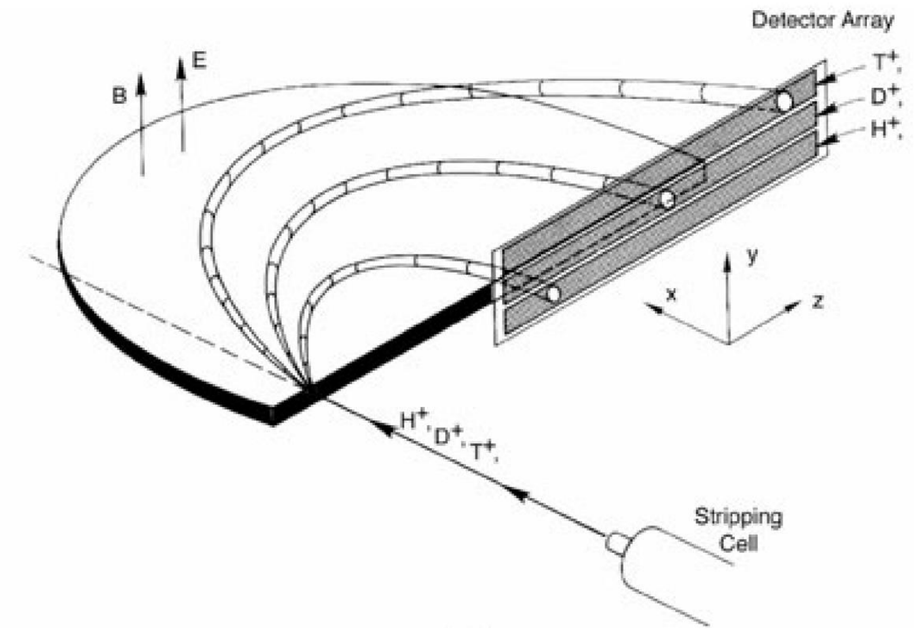
Can we tell energetic α-particles and energetic deuterium apart in fast-ion loss detector (FILD) and neutral particle analyzer (NPA) measurements?

1) FILD yes, NPA yes
2) FILD yes, NPA no
3) FILD no, NPA yes
4) FILD no, NPA no
5) I am not sure.

$$\boldsymbol{a} = \frac{q}{m}(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$



Fast-ion loss detector (FILD)



Neutral particle analyser (NPA)

Energetic particle diagnostics

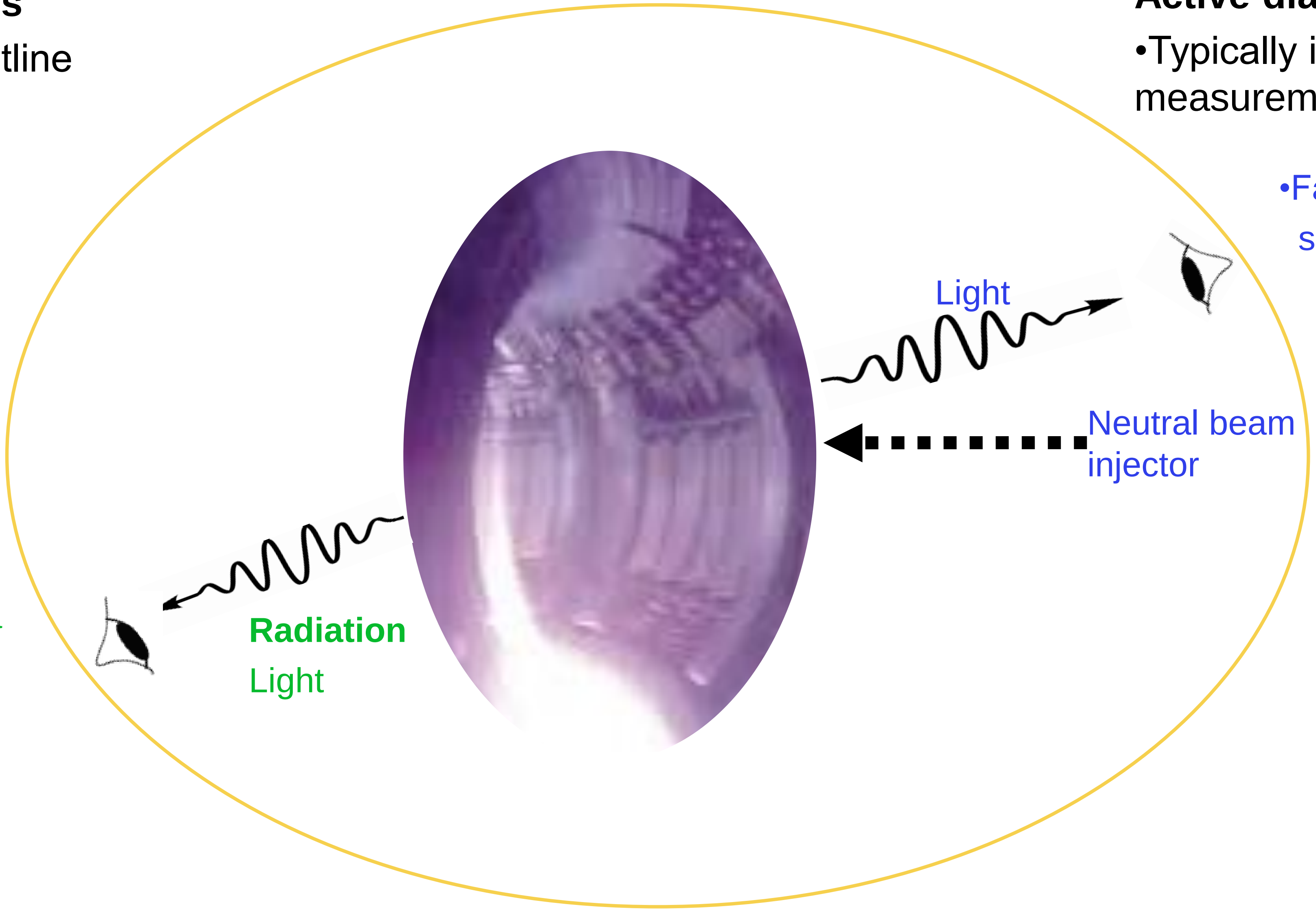
Passive diagnostics

- Typically along sightline

Active diagnostics

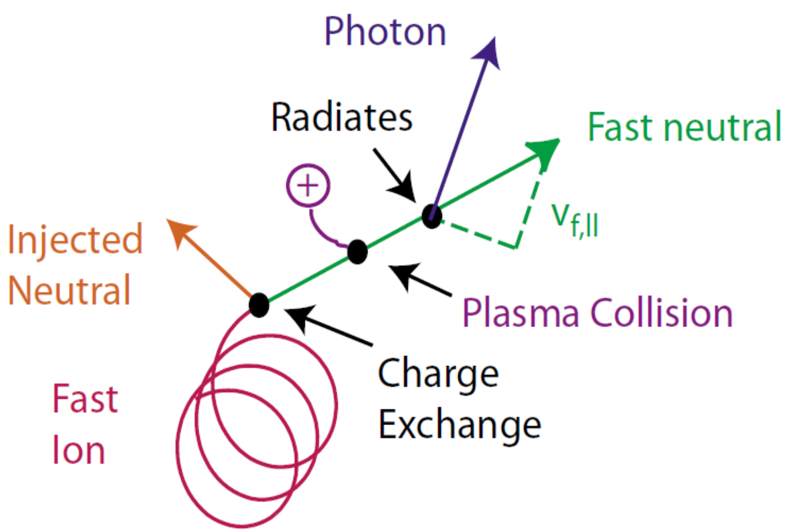
- Typically in a small measurement volume

- Fast-ion D-alpha spectroscopy

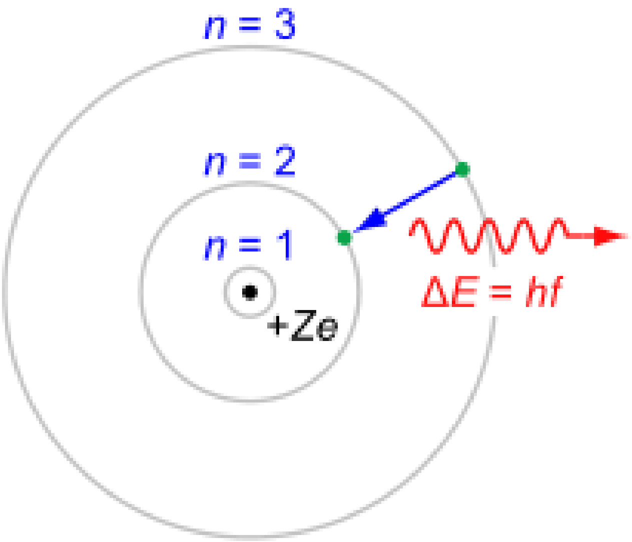


- Passive fast-ion D-alpha spectroscopy

Fast-ion D-alpha spectroscopy (FIDA)

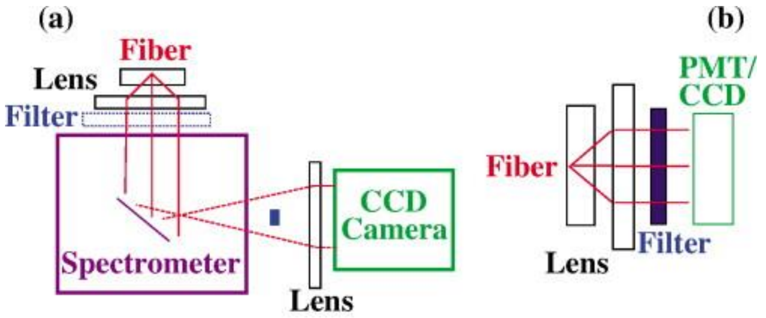


Bohr model of the deuterium atom



- Fast ions on helical trajectories
- Charge-exchange reaction with a neutral particle
- Fast ion is then neutral
- Electron transition 3→2 releases a D-alpha photon at 656.1 nm
- Doppler shift

$$\frac{u}{c} = \frac{\Delta f}{f} = -\frac{\Delta\lambda}{\lambda}$$

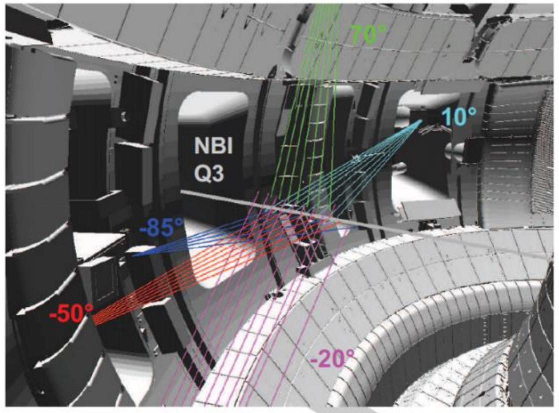


Active: Neutrals from neutral beam

Passive: Neutrals from elsewhere, plasma edge

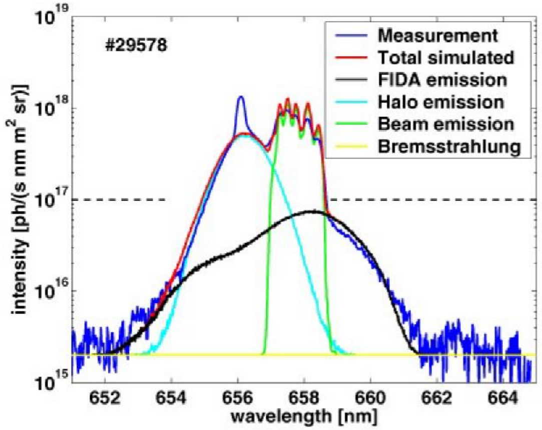
Heidbrink et al. (2004) PPCF, Heidbrink (2010) RSI

Fast-ion D-alpha (FIDA) spectroscopy at ASDEX Upgrade and DIII-D

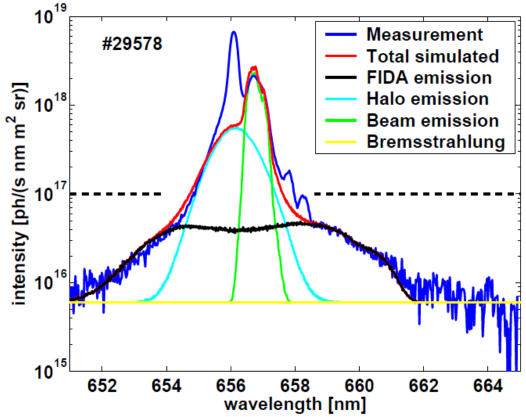


ASDEX Upgrade

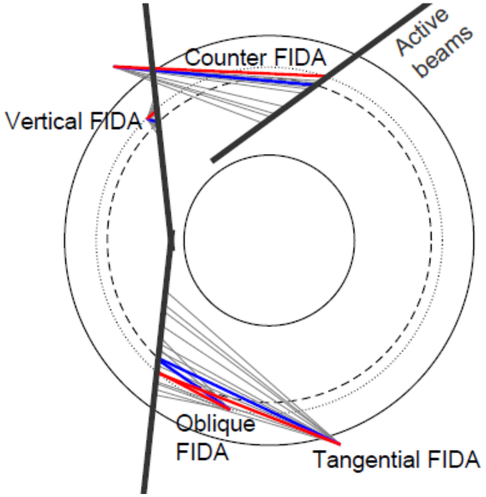
Tangential view, $\phi = 12°$



Perpendicular view, $\phi = 69°$

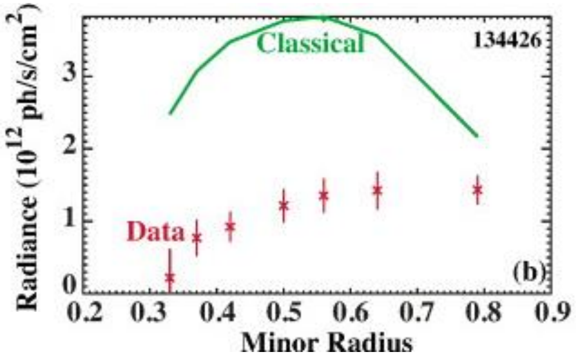


$$u = v_{\parallel} \cos\phi + v_{\perp} \sin\phi \cos\Gamma$$



DIII-D

Spatial FIDA emission profile



- Fast-ion D-alpha emission dominates at large Doppler-shifts
- Thermal-ion D-alpha emission (halo) dominates at small Doppler-shifts
- Direct D-alpha beam emission and impurity line radiation dominate in certain ranges
- Difficult if bremsstrahlung levels are high
- Background can be subtracted by NBI notch

Weiland et al. (2016) PPCF
Salewski et al. (2014) NF
Madsen et al. (2020) NF
Heidbrink (2010) RSI

Energy and momentum conservation imply the Doppler shift

- Fermi (1932) Rev. Mod. Phys. showed that energy and momentum conservation imply the Doppler shift
- Energy and momentum conservation equations:

$$\frac{1}{2}m_f v_f^2 + U = \frac{1}{2}m_f v_f'^2 + U' + E_{D\alpha}$$

$$m_f \boldsymbol{v}_f = m_f \boldsymbol{v}_f{}' + \boldsymbol{p}_{D\alpha}$$

- Isolate $\boldsymbol{v}_\mathrm{f}'$, square and substitute in energy:

$$\frac{1}{2}m_f v_f^2 + U = \frac{1}{2}m_f\left(v_f^2 + \frac{1}{m_f^2}p_{D\alpha}^2 - \frac{2}{m_f}\boldsymbol{v}_f \cdot \boldsymbol{p}_{D\alpha}\right) + U' + E_{D\alpha}$$

- Introduce $Q = U - U' = hf_0$ and $\boldsymbol{v}_f \cdot \boldsymbol{p}_{D\alpha} = u p_{D\alpha}$

$$hf_0 = \frac{1}{2m_f}p_{D\alpha}^2 - u p_{D\alpha} + E_{D\alpha}$$

- Introduce $p_{D\alpha} = \frac{E_{D\alpha}}{c}$ and $E_{D\alpha} \ll m_f c^2$

$$hf_0 = \frac{1}{2m_f c^2}E_{D\alpha}^2 - \frac{u}{c}E_{D\alpha} + E_{D\alpha} \approx \left(1 - \frac{u}{c}\right)hf$$

- Taylor expansion gives the usual Doppler shift formula:

$$\frac{\Delta f}{f_0} = \frac{u}{c} \qquad \frac{\Delta\lambda}{\lambda_0} = -\frac{u}{c}$$

- Doppler shift is proportional to projected velocity.

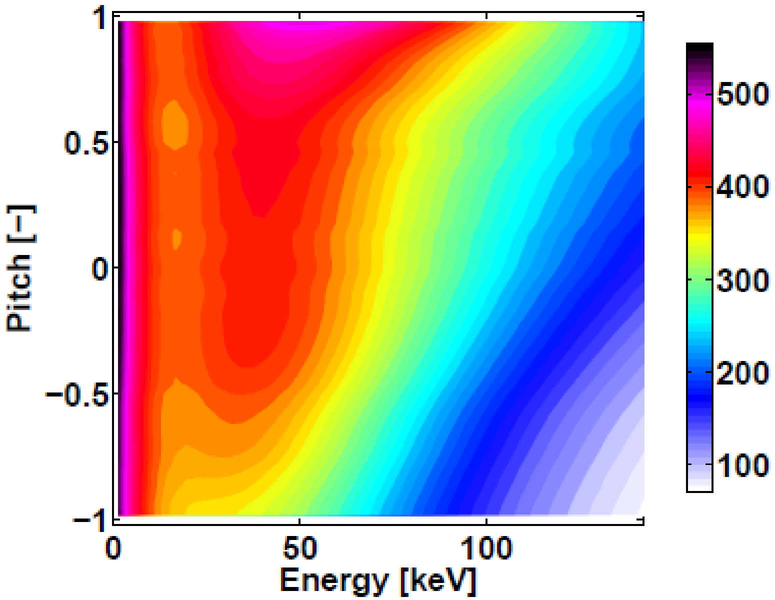
$$u = v_\parallel \cos\phi + v_\perp \sin\phi \cos\Gamma$$

Fast-ion D-alpha (FIDA) spectroscopy– spectrum formation

- Doppler shift $\lambda = \lambda_0 \left(1 + \frac{u}{c}\right)$

- Stark splitting $\lambda = (\lambda_0 + s_l|\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}|)\left(1 + \frac{u}{c}\right)$ Split in 15 lines with different coefficients s_l

- Charge-exchange probabilities
- Electron transition probabilities



- D-alpha emission at any wavelength per ion for a 60 keV beam
- Depends only on charge-exchange and electron transition probabilities
- Little D-alpha emission energies 100-150 keV larger than beam injection energy

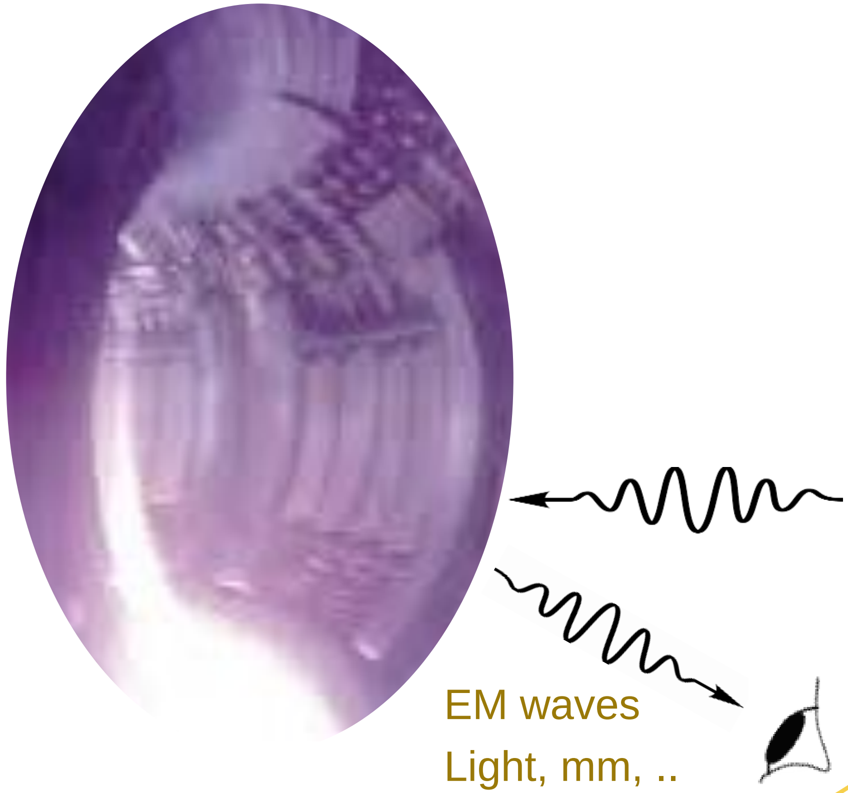
- Instrumental broadening

Salewski et al (2014) PPCF

Energetic particle diagnostics

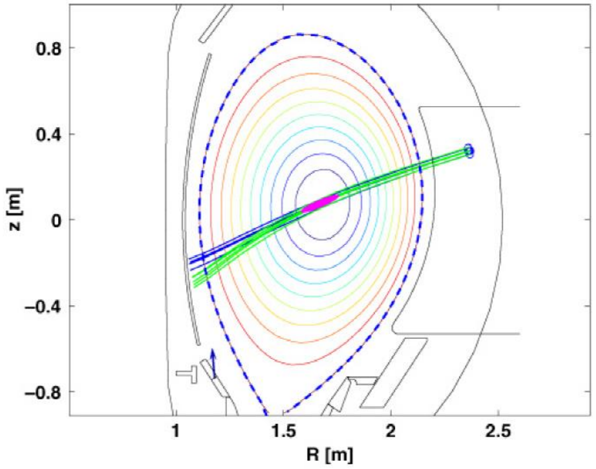
Active diagnostics

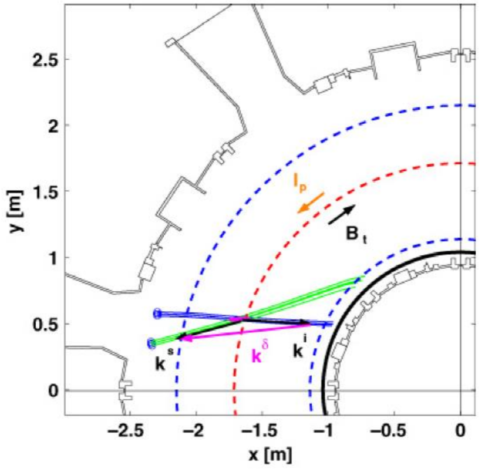
- Typically in a small measurement volume



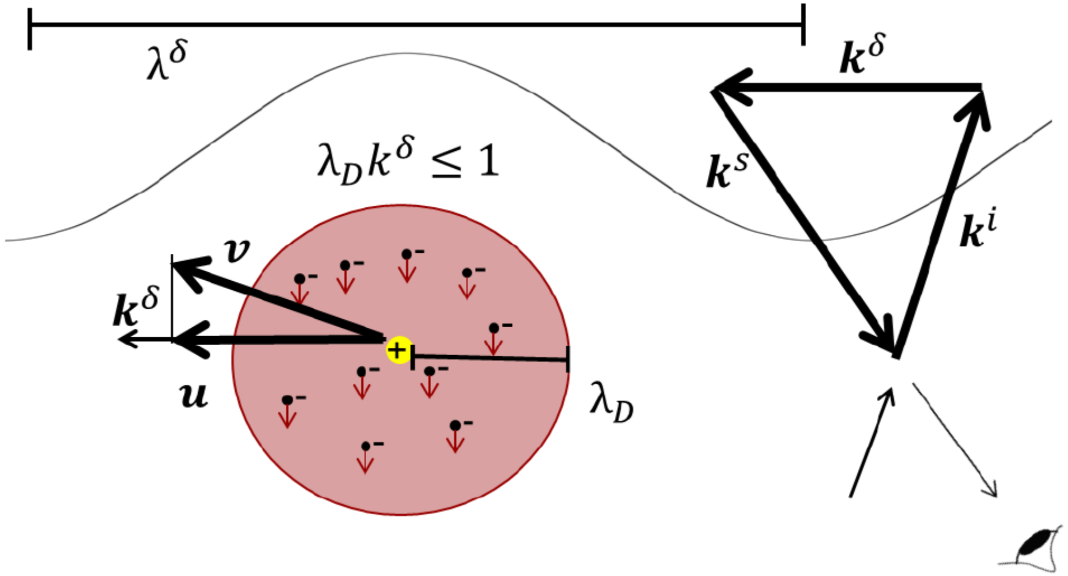
- Collective Thomson scattering

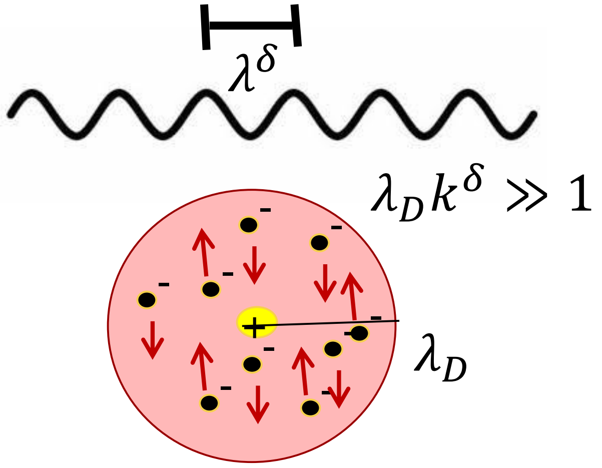
Collective Thomson scattering (CTS)





- Monochromatic gyrotron probe radiation ($\boldsymbol{k}^i$) overlaps the acceptance cone of a receiver beam ($\boldsymbol{k}^s$)
- Gyrotron power modulated to subtract electron cyclotron emission (ECE) background
- Electromagnetic waves always interact with electrons due to the lower mass.
- Resolves fluctuation wave vector $\boldsymbol{k}^\delta = \boldsymbol{k}^s - \boldsymbol{k}^i$





- Collective Thomson scattering: Debye sphere is small compared to the fluctuation wavelength

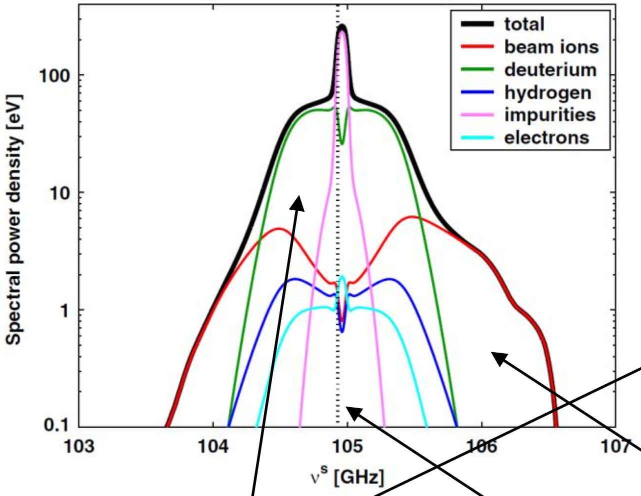
$$\lambda_D k^\delta \leq 1$$

Salewski et al. (2010) NF, Salewski (2020) dr. thesis

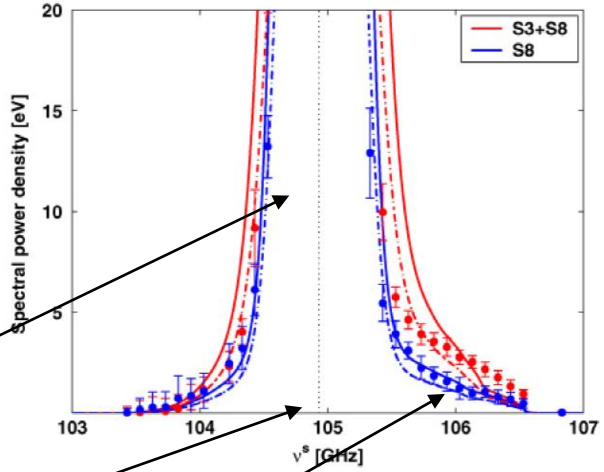
Collective Thomson scattering (CTS)

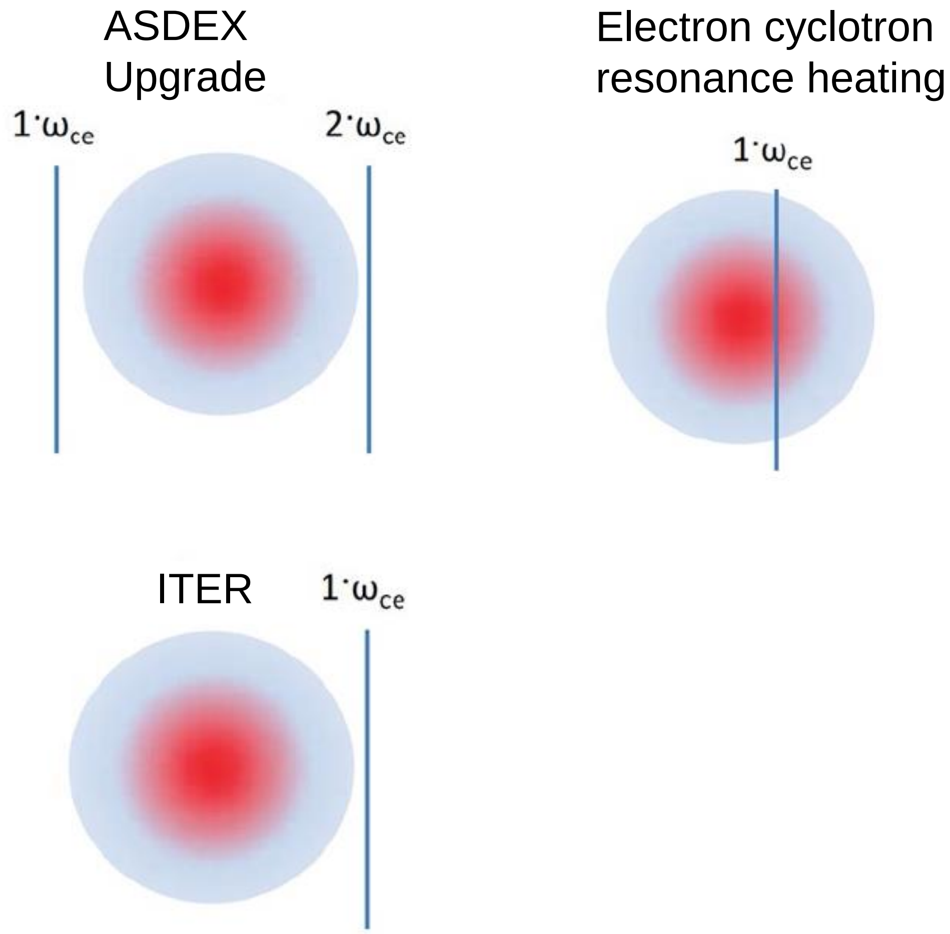
- Unabsorbed gyrotron probe radiation scattered in the plasma

Synthetic CTS spectrum



Measurement and simulation for 1 and 2 NBIs





Thermal and fast ions can cause low frequency shifts

Gyrotron frequency

Only fast ions can cause high frequency shifts.

Salewski et al. (2010) NF

Moseev et al. (2018) Rev. Mod. Plasma Phys.

Collective Thomson scattering (CTS)

- Energy and momentum conservation in the dressed particle model

$$\hbar\omega^i + \frac{1}{2} m_f v_f^2 = \hbar\omega^s + \frac{1}{2} m_f v_f'^2$$

$$\hbar\boldsymbol{k}^i + m_f \boldsymbol{v}_f = \hbar\boldsymbol{k}^s + m_f \boldsymbol{v}_f'$$

- Isolate $\boldsymbol{v}_f$' and square using $\boldsymbol{k}^\delta = \boldsymbol{k}^s - \boldsymbol{k}^i$

$$v_f'^2 = v_f^2 - 2\frac{\hbar}{m_f}\boldsymbol{v}_f \cdot \boldsymbol{k}^\delta + \frac{\hbar^2}{m_f^2}\left(k^\delta\right)^2$$

- Substitute into energy, using $\omega^\delta = \omega^s - \omega^i$

$$\omega^\delta = \boldsymbol{v}_f \cdot \boldsymbol{k}^\delta - \frac{\hbar}{2m_f}\left(k^\delta\right)^2 \approx \boldsymbol{v}_f \cdot \boldsymbol{k}^\delta$$

- The projected velocity u appears

$$\omega^\delta = \boldsymbol{v}_f \cdot \boldsymbol{k}^\delta = u k^\delta$$

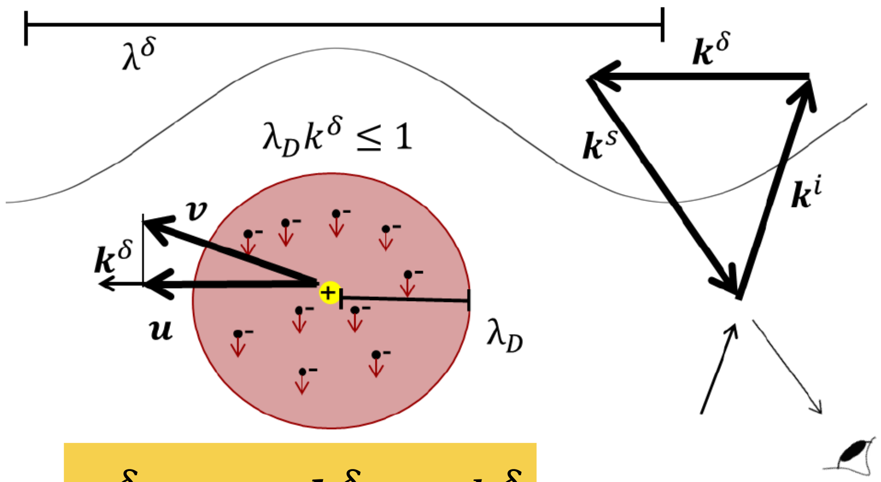
- Doppler shift is proportional to projected velocity.

$$u = v_\parallel \cos\phi + v_\perp \sin\phi \cos\Gamma$$

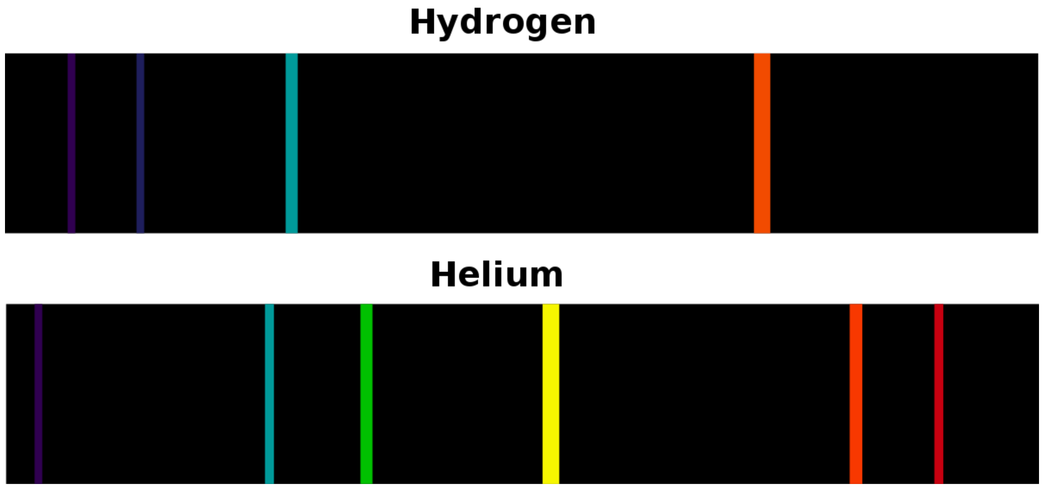
Can we tell energetic α-particles and energetic deuterium apart in fast-ion charge exchange spectroscopy (FICX) and collective Thomson scattering (CTS) measurements?

1) FICX yes, CTS yes
2) FICX yes, CTS no
3) FICX no, CTS yes
4) FICX no, CTS no
5) I am not sure.

Fast-ion charge exchange spectroscopy (FICX) is the same as fast-ion D-alpha (FIDA) spectroscopy, but on any emission line.



$$\omega^\delta = \boldsymbol{v}_f \cdot \boldsymbol{k}^\delta = u k^\delta$$



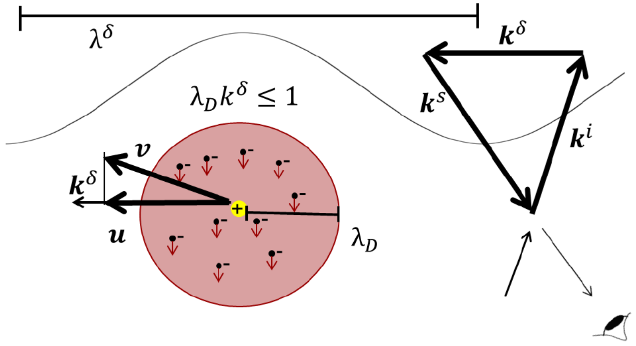
Can we tell energetic α-particles and energetic deuterium apart in fast-ion charge exchange spectroscopy (FICX) and collective Thomson scattering (CTS) measurements?

1) FICX yes, CTS yes
2) FICX yes, CTS no
3) FICX no, CTS yes
4) FICX no, CTS no
5) I am not sure.

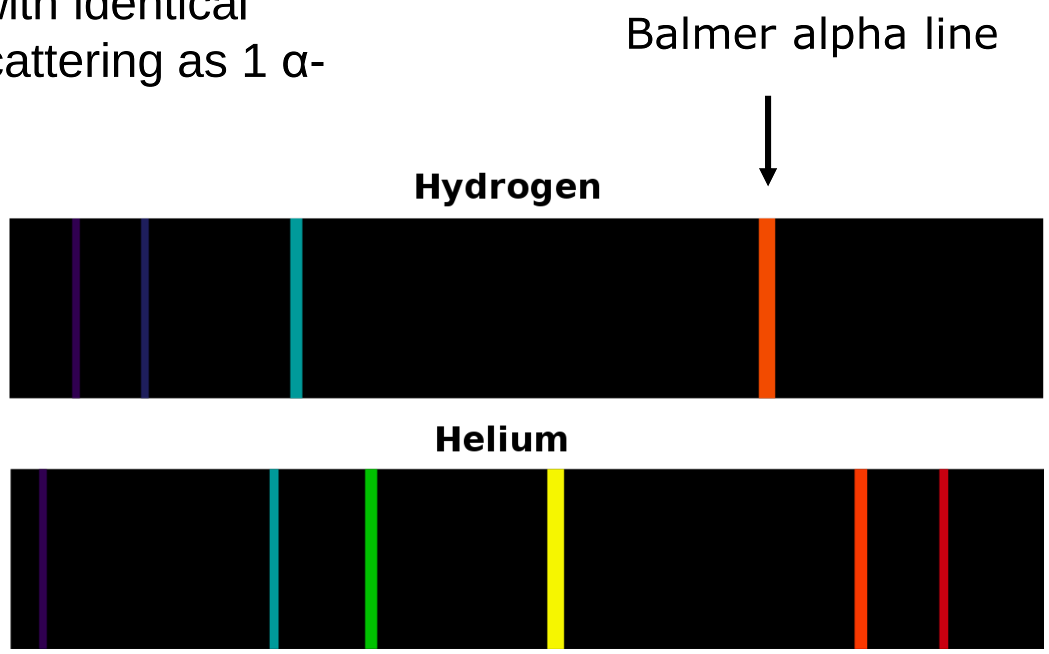
In FIDA/FICX, we know the emitting species from the detected wavelength.

In CTS, 2 deuterium ions with identical velocity cause the same scattering as 1 α-particle.

$$\boldsymbol{a} = \frac{q}{m}(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$



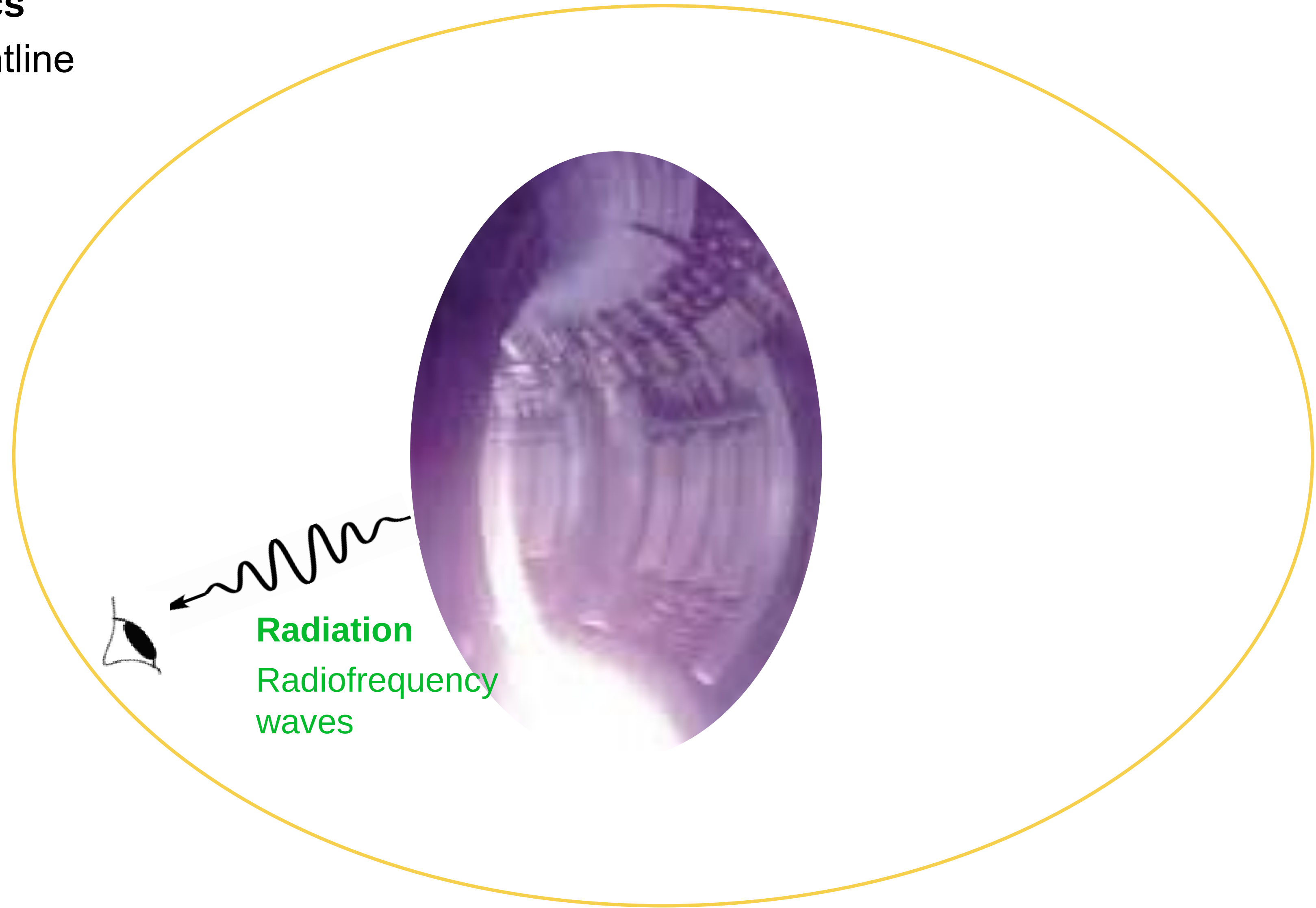
$$\omega^\delta = \boldsymbol{v}_f \cdot \boldsymbol{k}^\delta = u k^\delta$$



Energetic particle diagnostics

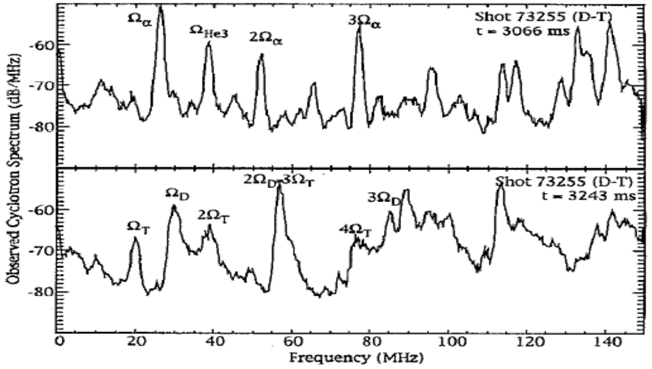
Passive diagnostics

•Typically along sightline



•Ion cyclotron emission spectroscopy

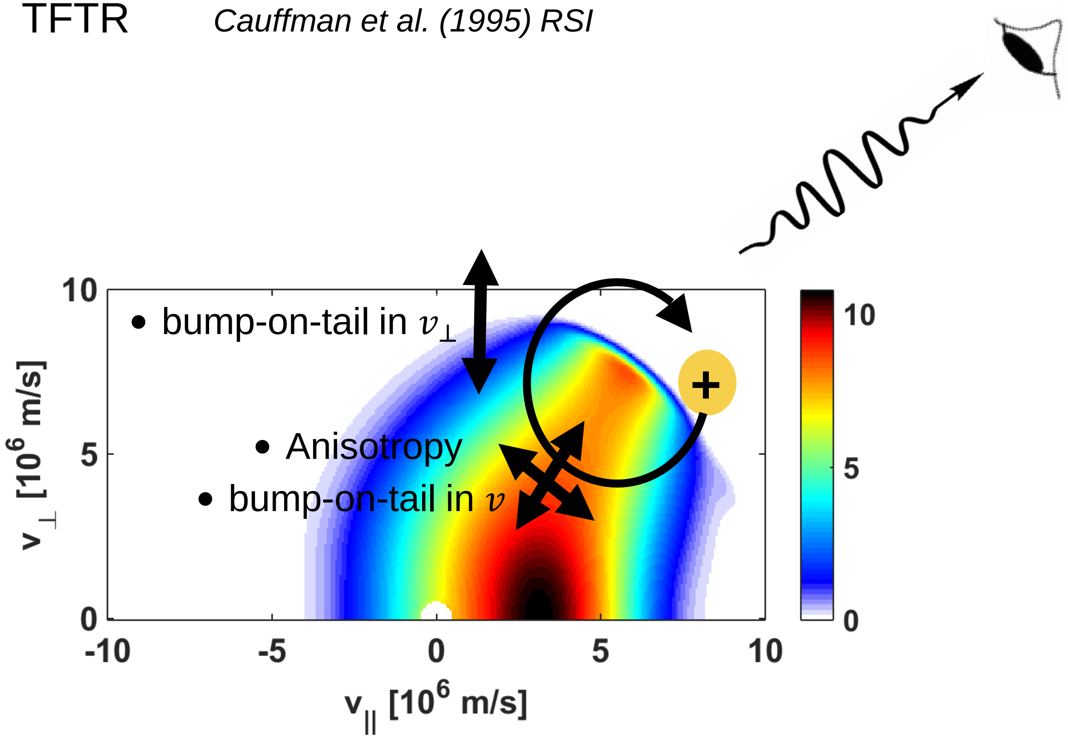
Ion cyclotron emission (ICE) spectroscopy

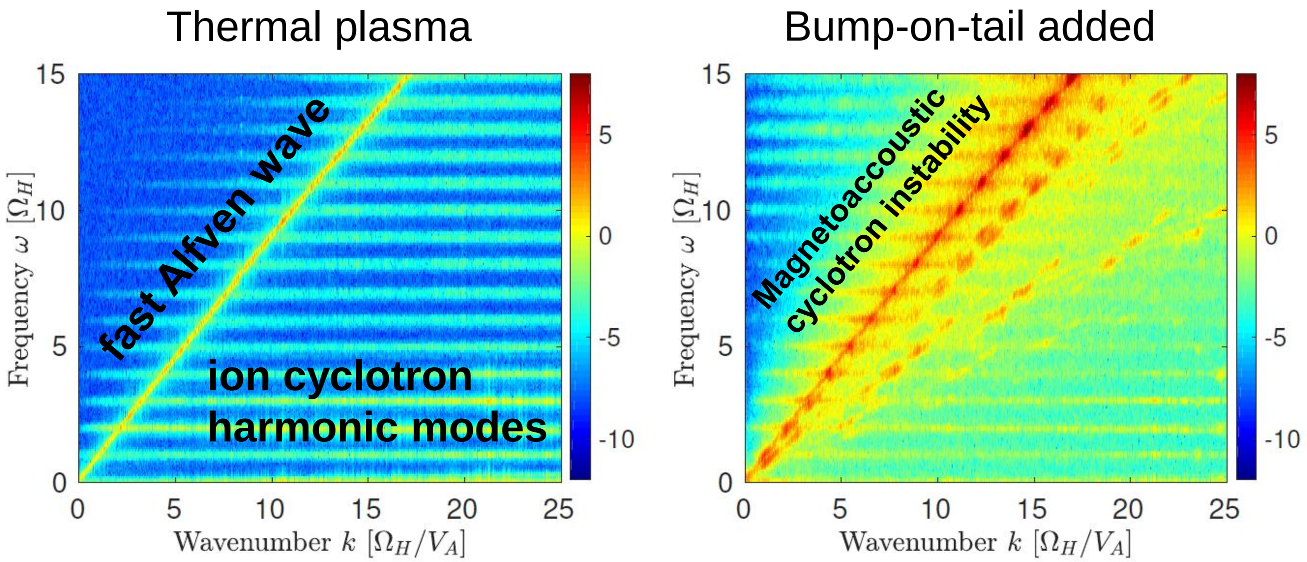


TFTR *Cauffman et al. (1995) RSI*

- Magnetoacoustic cyclotron instability and/or geometry, e.g. compressional Alfven eigenmodes
- Resonance between fast Alfven wave and fast ion cyclotron harmonic modes $\omega = kv_A = n\omega_c$

Energetic particles can drive instabilities for
- Inhomogeneity (spatial gradients)
- Deviation from a Maxwellian distribution
  - bump-on-tail in $v_\perp$ or v
  - anisotropy





Reman et al. (2019) NF

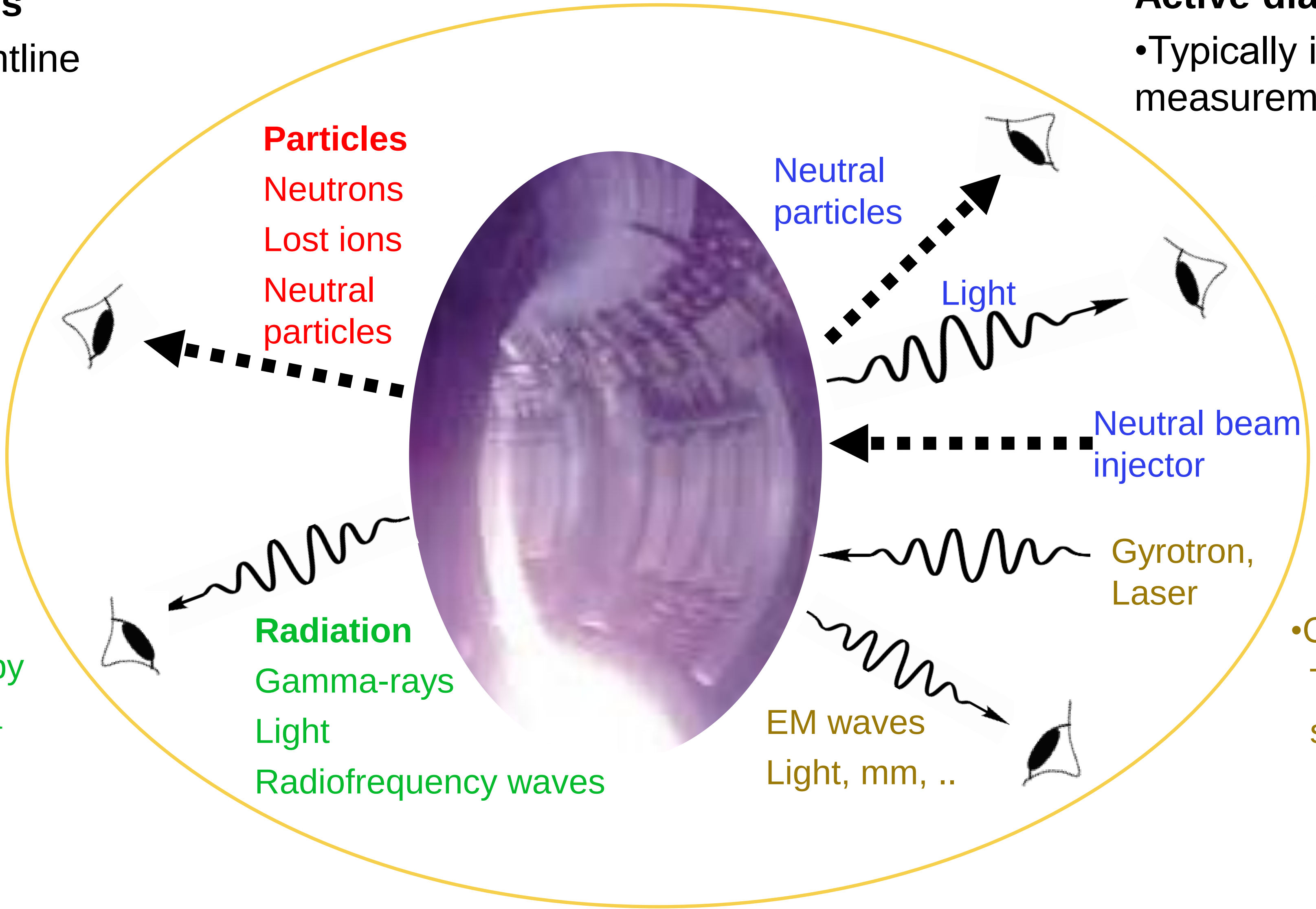
Energetic particle diagnostics

Passive diagnostics

- Typically along sightline

- Neutron counter
- Neutron camera
- Neutron emission spectroscopy
- Fast-ion loss detector
- Charged fusion product detector
- Passive neutral particle analyser

- Gamma-ray camera,
- Gamma-ray spectroscopy
- Passive fast-ion D-alpha spectroscopy
- Ion cyclotron emission spectroscopy



Active diagnostics

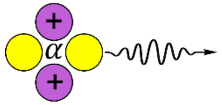
- Typically in a small measurement volume

- Neutral particle analyser
- Imaging neutral particle analyzer
- Fast-ion D-alpha spectroscopy

- Collective Thomson scattering

Outline

- Introduction: Why diagnose energetic particles?
- Physics of energetic particle diagnostics



- **The forward problem: Spectrum formation for energetic particle diagnostics**
- The inverse problem: Inferring energetic particle distributions from diagnostic data
- Summary

Energy and momentum conservation, spectrum formation and the projected velocity

Neutron emission spectroscopy (NES)

$$\frac{1}{2}m_f v_f^2 + \frac{1}{2}m_r v_r^2 + Q = \frac{1}{2}m_{pr} v_{pr}^2 + \frac{1}{2}m_n v_n^2$$

$$m_f \boldsymbol{v}_f + m_r \boldsymbol{v}_r = m_{pr} \boldsymbol{v}_{pr} + m_n \boldsymbol{v}_n$$

$$E_n \approx \frac{m_{pr}}{m_{pr}+m_n} Q + \frac{m_{pr}-m_f}{m_{pr}+m_n}\frac{1}{2} m_f v_f^2 + \frac{m_f m_n}{m_{pr}+m_n} u v_n$$

Gamma-ray spectroscopy (GRS), one-step reaction

$$\frac{1}{2}m_f v_f^2 + \frac{1}{2}m_r v_r^2 + Q = \frac{1}{2}m_{pr} v_{pr}^2 + E_\gamma$$

$$m_f \boldsymbol{v}_f + m_r \boldsymbol{v}_r = m_{pr} \boldsymbol{v}_{pr} + \boldsymbol{p}_\gamma$$

$$E_\gamma \approx \left(1 + \frac{m_f}{m_{pr}}\frac{u}{c}\right)\left(Q + \left(1 - \frac{m_f}{m_{pr}}\right) E_f\right)$$

$$u = v_\parallel \cos\phi + v_\perp \sin\phi \cos\Gamma$$

Fast-ion D-alpha (FIDA) spectroscopy

$$\frac{1}{2}m_f v_f^2 + U = \frac{1}{2}m_f v_f'^2 + U' + E_{D\alpha}$$

$$m_f \boldsymbol{v}_f = m_f \boldsymbol{v}_f{}' + \boldsymbol{p}_{D\alpha}$$

$$E_{D\alpha} \approx \frac{hc}{\lambda_0}\left(1 + \frac{u}{c}\right) \qquad \frac{\Delta\lambda}{\lambda_0} \approx -\frac{u}{c}$$

Collective Thomson scattering

$$\hbar\omega^i + \frac{1}{2}m_f v_f^2 = \hbar\omega^s + \frac{1}{2}m_f v_f'^2 \qquad \omega^\delta = \omega^s - \omega^i$$

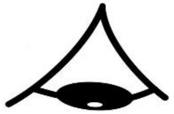
$$\hbar\boldsymbol{k}^i + m_f \boldsymbol{v}_f = \hbar\boldsymbol{k}^s + m_f \boldsymbol{v}_f' \qquad \boldsymbol{k}^\delta = \boldsymbol{k}^s - \boldsymbol{k}^i$$

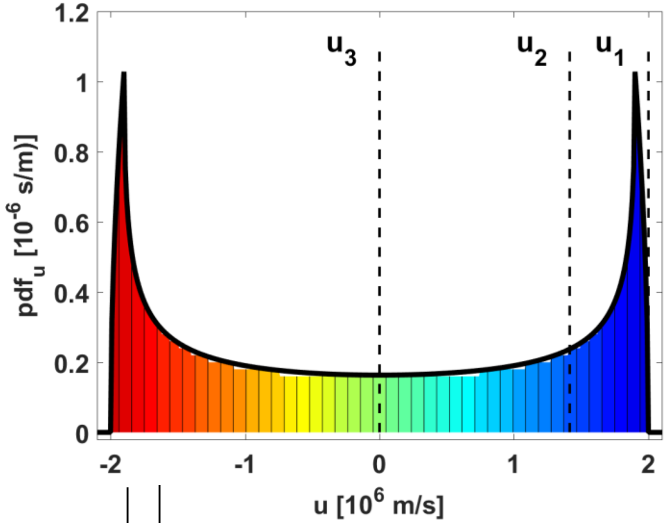
$$E_{\omega^\delta} \approx \hbar \boldsymbol{v}_f \cdot \boldsymbol{k}^\delta = \hbar u k^\delta \qquad \omega^\delta \approx \boldsymbol{v}_f \cdot \boldsymbol{k}^\delta = u k^\delta$$

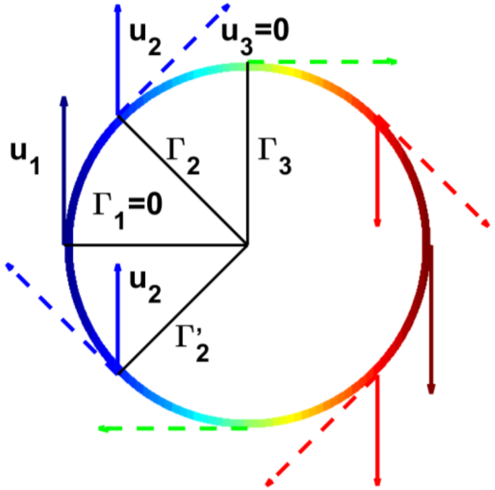
The swinging 258 Hz tuning fork

Processes with Doppler shifted signals with known frequency

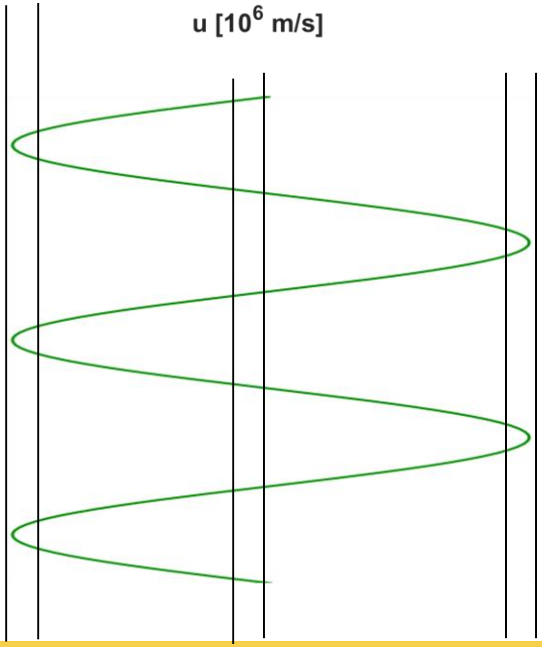
- Gamma-ray emission spectroscopy
- Neutron emission spectroscopy
- Scattering of waves
- D-alpha emission







$$\frac{u}{c} = \frac{\Delta f}{f}$$

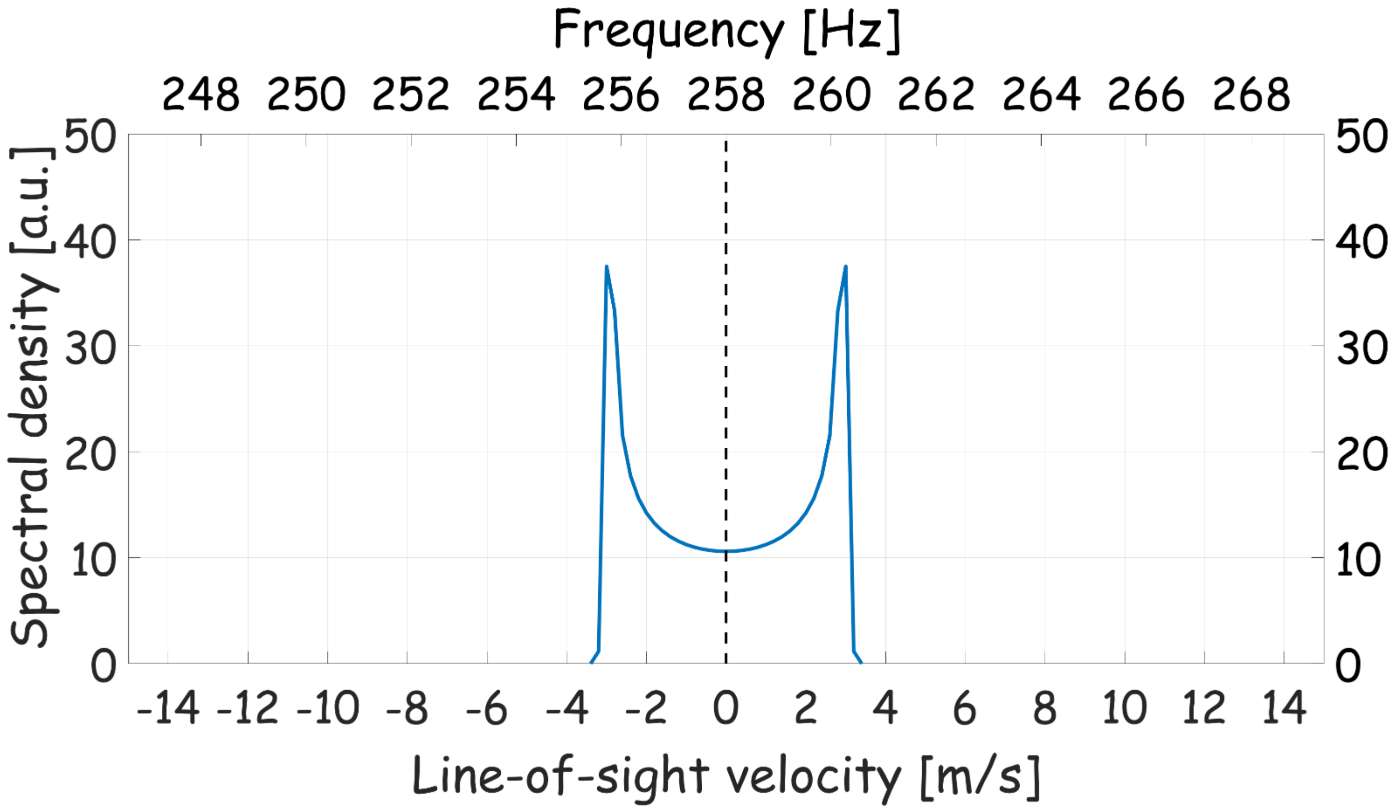


Doppler-shifted sound from a 258 Hz tuning fork.

How fast am I swinging the 258 Hz tuning fork?

- You record the sound spectrum of somebody swinging a 258 Hz tuning fork.
- What is the swing speed?

$$\frac{u}{c} = \frac{\Delta f}{f}$$

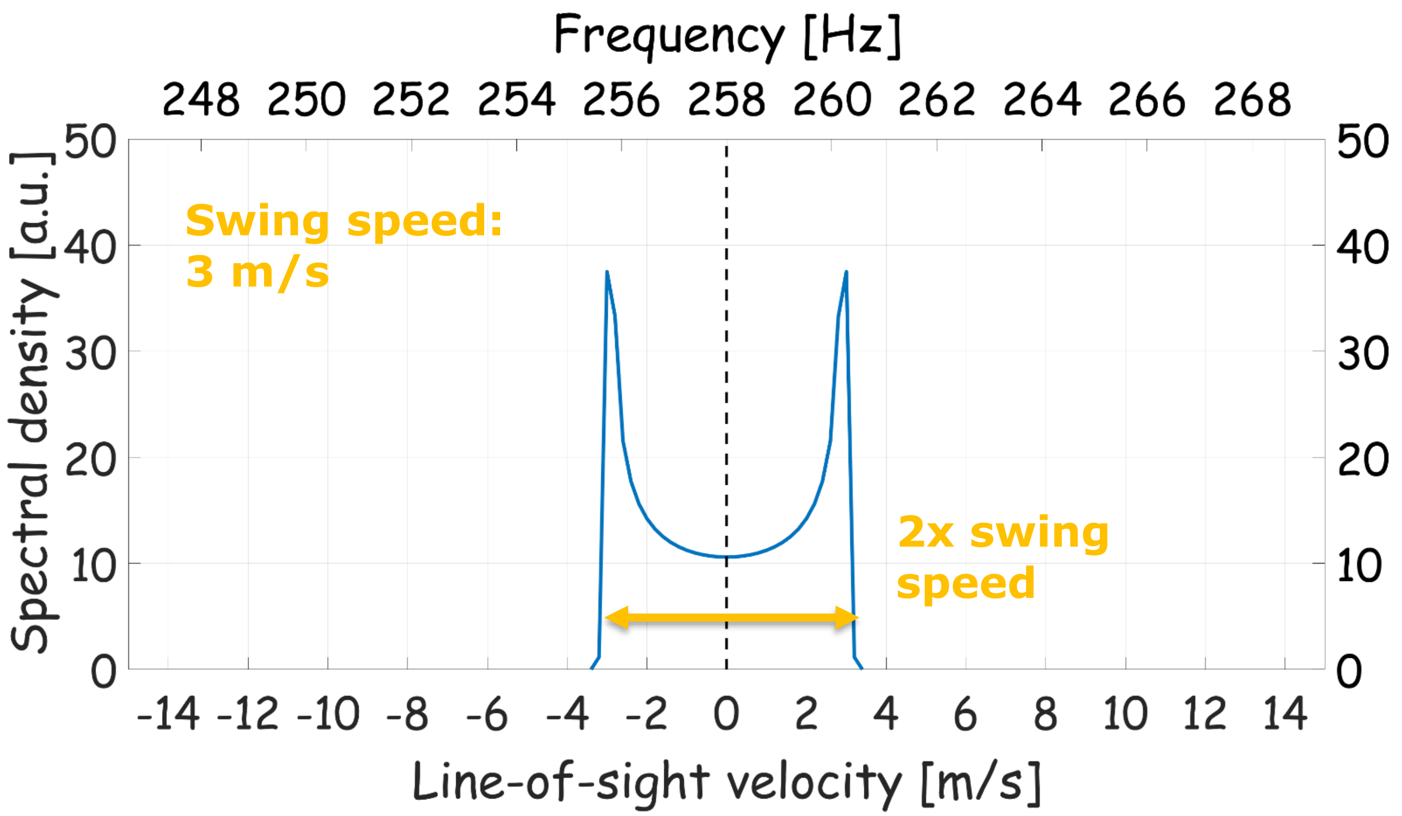


How fast am I swinging the 258 Hz pitchfork?

- You record the sound spectrum of somebody swinging a 258 Hz tuning fork.

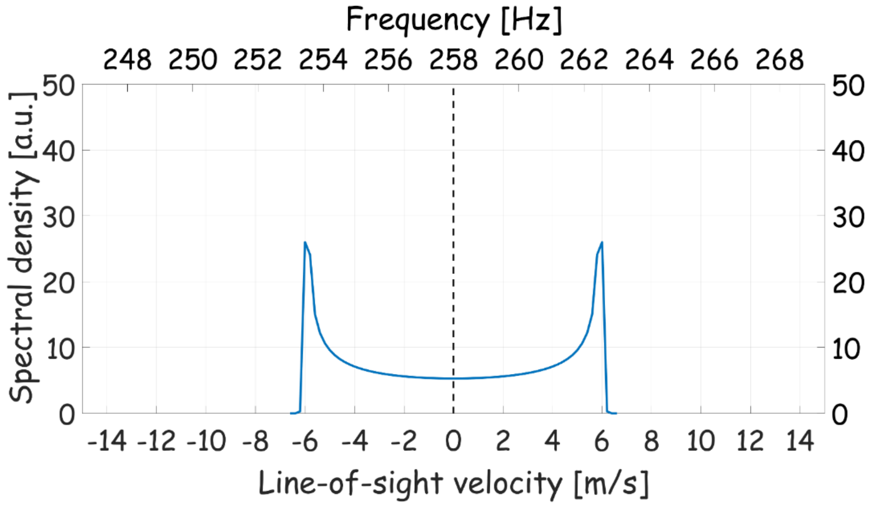
What is the swing speed?

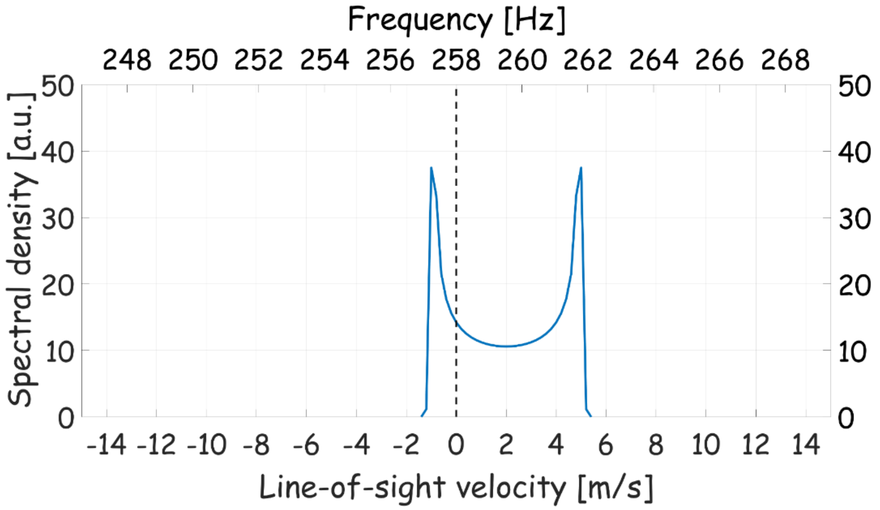
$$\frac{u}{c} = \frac{\Delta f}{f}$$



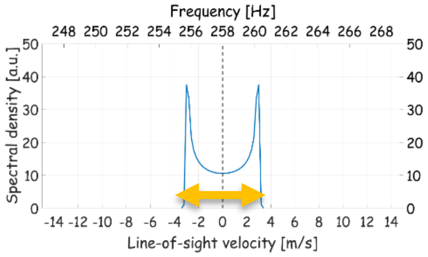
How fast am I walking and swinging the 258 Hz tuning fork?

- You record the sound spectrum of somebody walking and swinging a 258 Hz tuning fork.
- What is the swing speed in each case?
- What is the walk speed in each case?





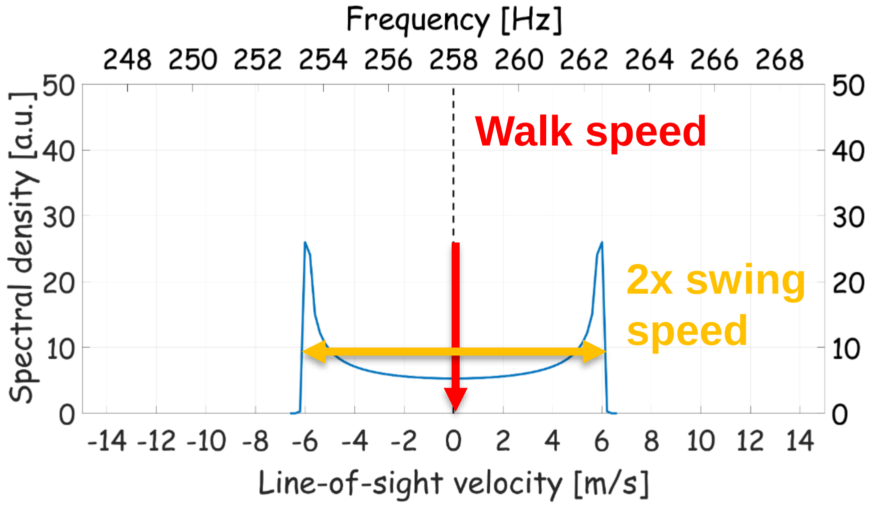
Previous solution for swing speed: 3 m/s



2x swing speed

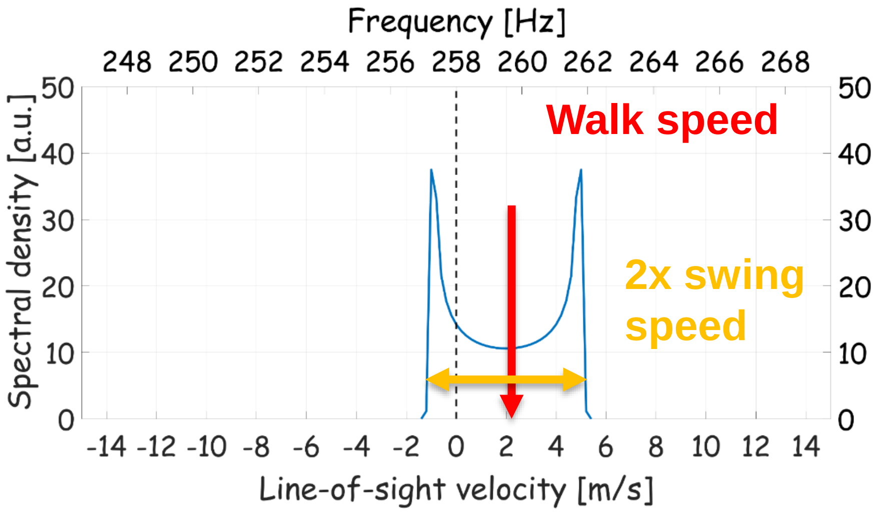
How fast am I walking and swinging the 258 Hz tuning fork?

- You record the sound spectrum of somebody walking and swinging a 258 Hz tuning fork.
- What is the swing speed in each case?
- What is the walk speed in each case?



Walk speed: 0 m/s

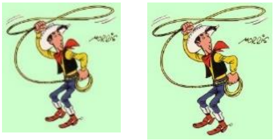
Swing speed: 6 m/s



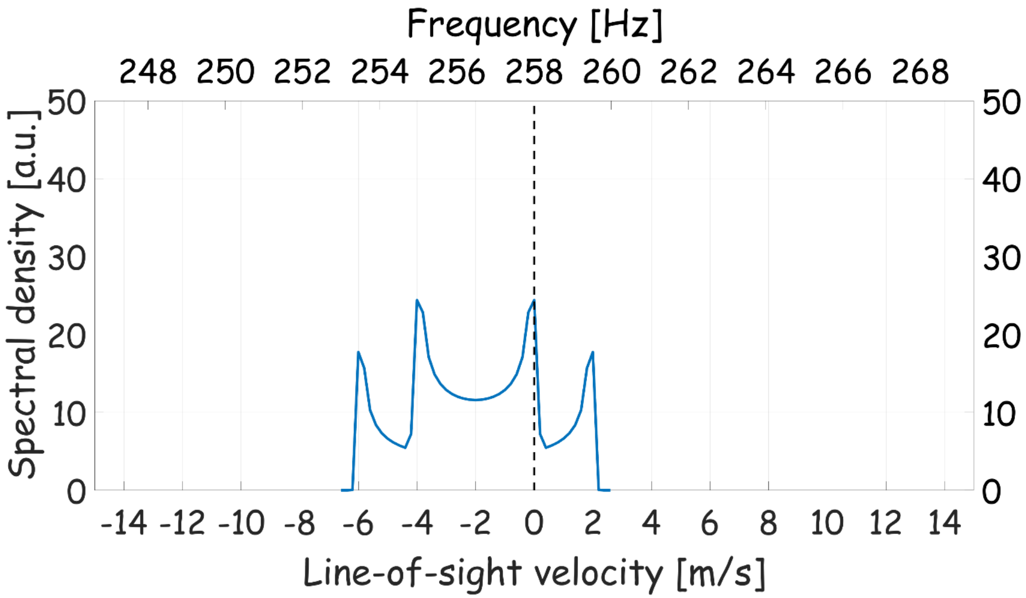
Walk speed: 2 m/s

Swing speed: 3 m/s

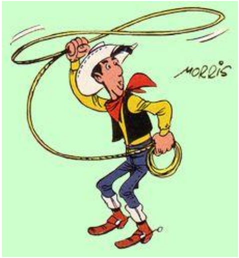
How fast are 2 people walking and swinging 258 Hz tuning forks?

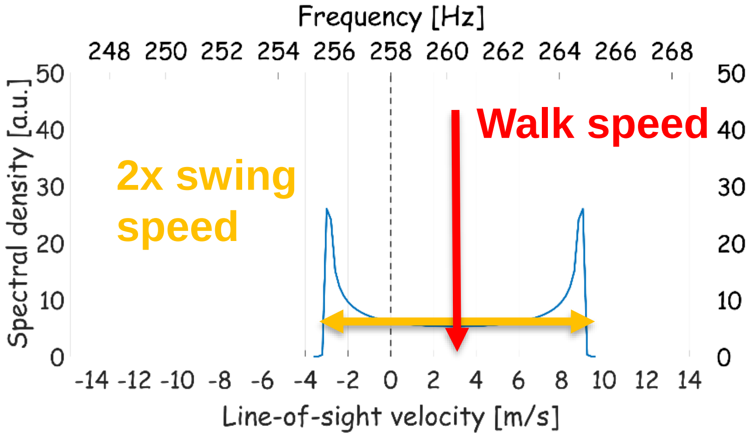


2 people

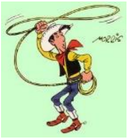


Previous solution for 1 person

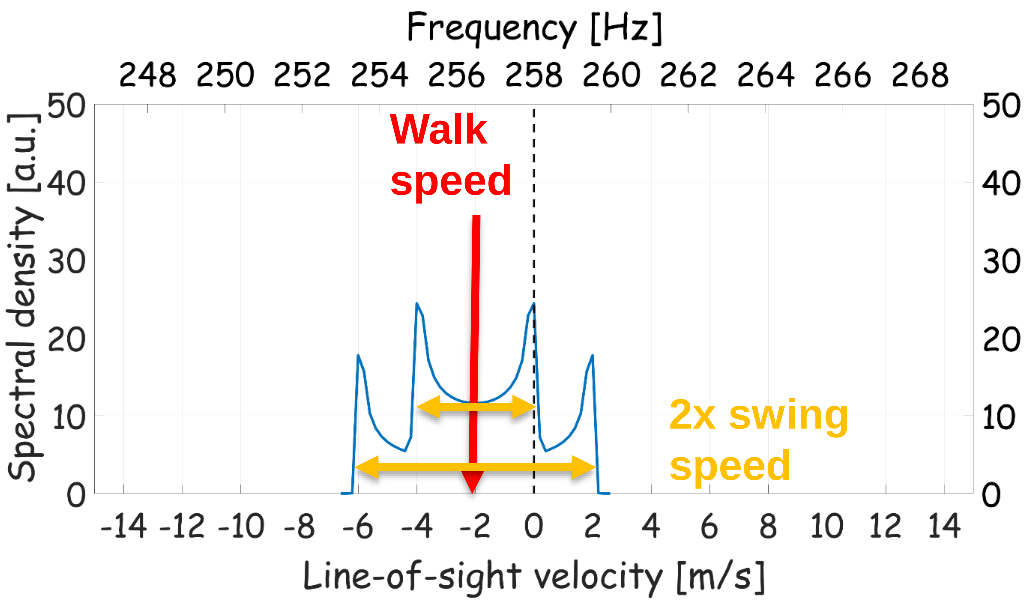




How fast are 2 people walking and swinging 258 Hz tuning forks?

2 people



Is this the solution?

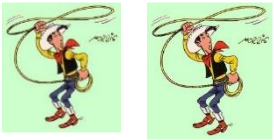
Both persons walk away at 2 m/s

One person swings 2 m/s

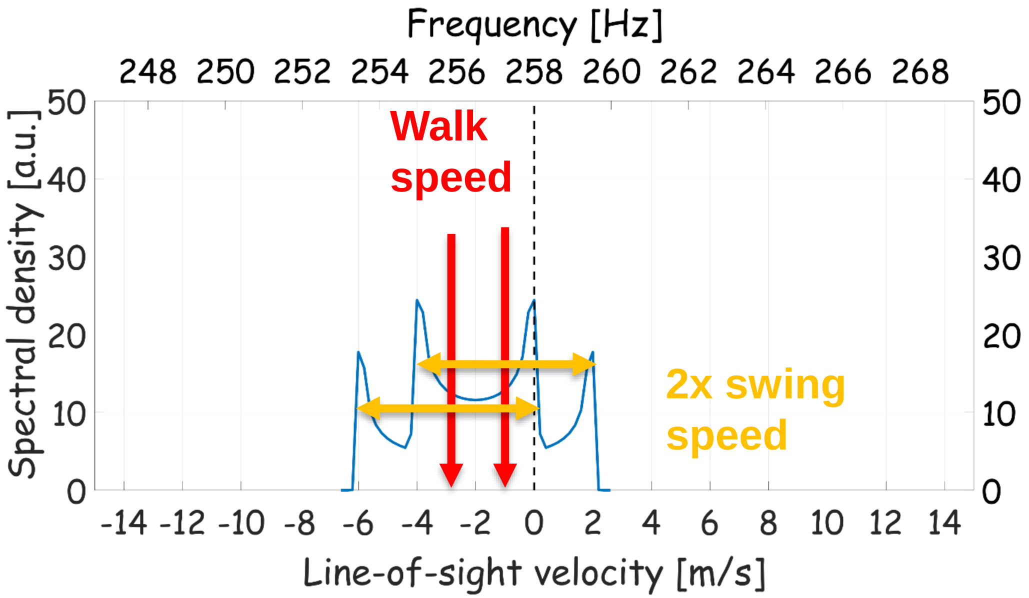
One person swings 4 m/s

DTU

How fast are 2 people walking and swinging 258 Hz tuning forks?



2 people



~~**Is this the solution?**~~

~~Both persons walk away at 2 m/s~~

~~One person swings 2 m/s~~

~~One person swings 4 m/s~~

Correct solution:

One person walks away at 1 m/s

One person walks away at 3 m/s

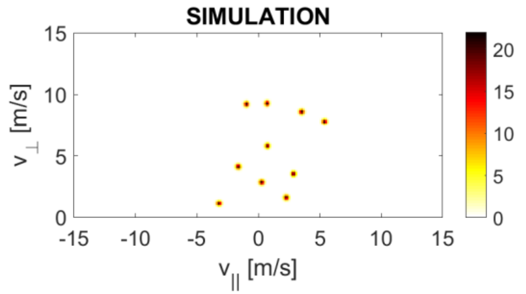
Both persons swing 3 m/s

- Difficult to see by eye
- Easy to solve with a computer by least square fitting

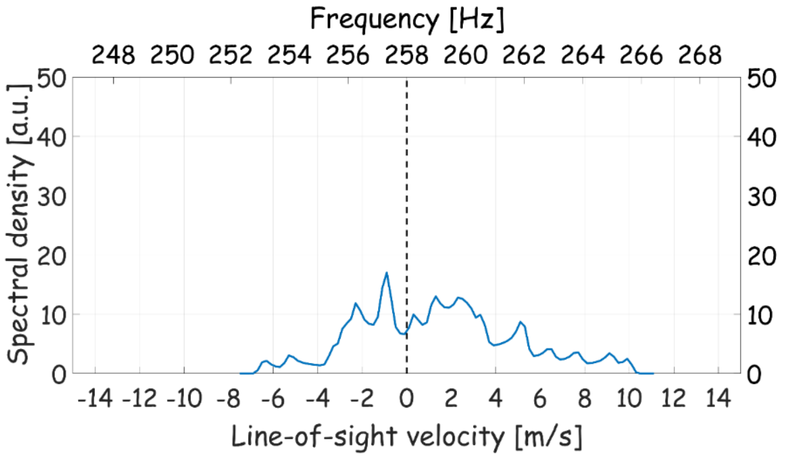
How fast are 10 people walking and swinging 258 Hz tuning forks? Or 100? Or 10.000?

10 people

Swing speed



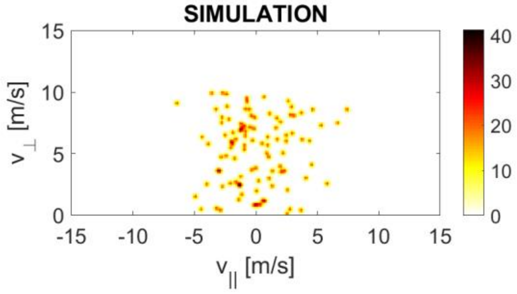
Walk speed



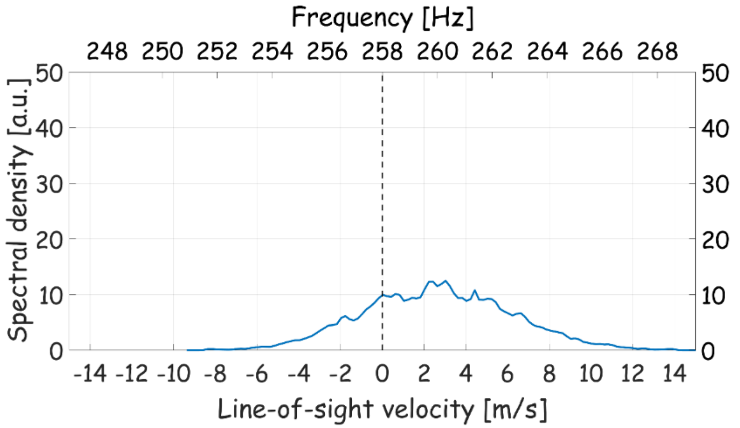
- Can find velocities by least square fitting

100 people

Swing speed

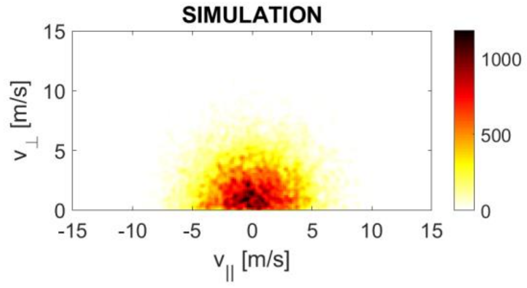


Walk speed

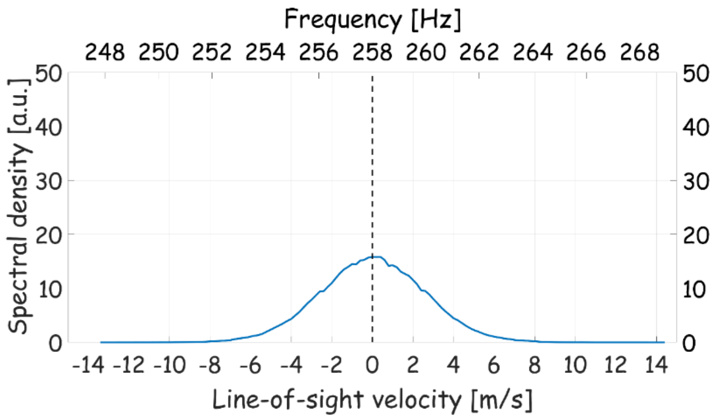


10000 people

Swing speed



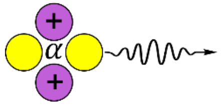
Walk speed



- Least-square fitting fails
- Need tomography-type inversion methods

Outline

- Introduction: Why diagnose energetic particles?
- Physics of energetic particle diagnostics
- The forward problem: Spectrum formation for energetic particle diagnostics



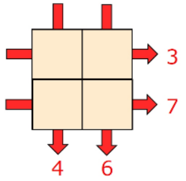
- **The inverse problem: Inferring energetic particle distributions from diagnostic data**
- Summary

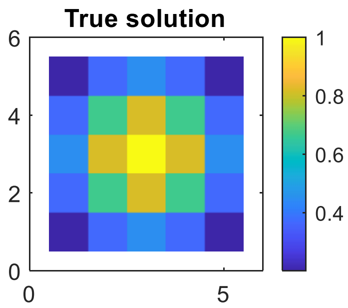
From the back-of-the-envelope towards typical plasma tomography

- Determine 5x5=25 unknowns from 5 column sums and 5 row sums
- True solution: 2D Gaussian
- Calculate sums, add 10% noise

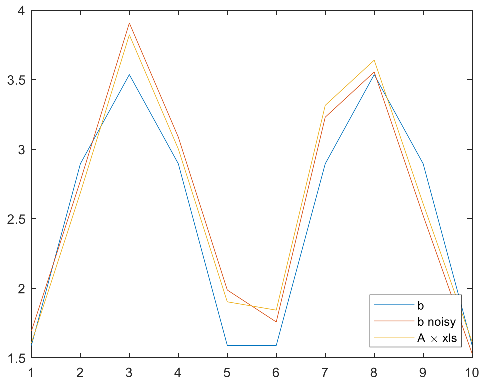
- Inverse problem: find *F*, given noisy *S*.
- For *W* with full rank, a least square fit is found by the 'normal equations'

- Almost always in tomography, *W* is rank-deficient and the inverse problem is ill-posed.
- **Ill-posed**: small change in *S* leads to large change in *F*.
- Measurement noise leads to random jitter in the 2D image.

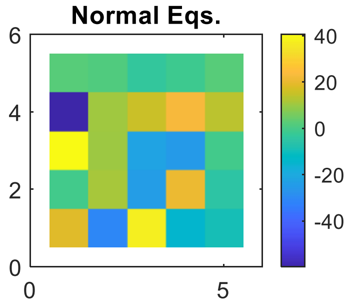




$$S = WF$$

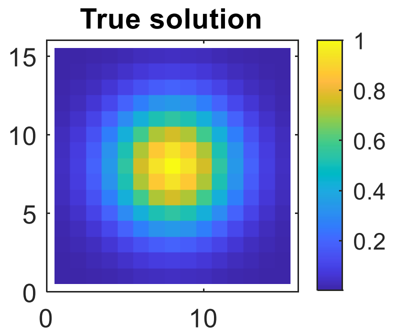


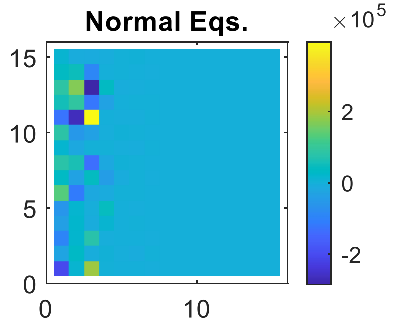
$$F = (W^T W)^{-1} W^T S$$



$$S = WF$$

- The same problem with 15x15=225 unknowns from 15 column sums and 15 row sums doesn't work, either.





Tikhonov regularization: Penalize undesired features

Tikhonov expressed prior information as a penalty term

minimize $\left\| \binom{W}{\lambda L} F - \binom{S}{0} \right\|_2$

Regularization parameter λ: balance between data fitting residual and penalty term

0^{th} order Tikhonov : L is the identity matrix which favours solutions without large spikes

1^{st} order Tikhonov : L is a gradient operator matrix which favours solutions without large gradients

…

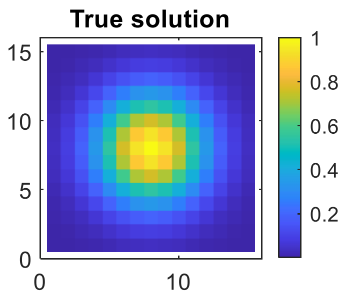
The normal equations of the Tikhonov problem are →

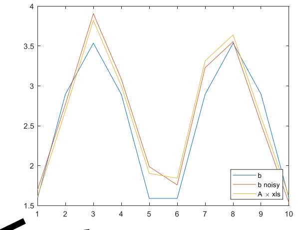
$$F_\lambda = (W^T W + \lambda^2 L^T L)^{-1} W^T S$$

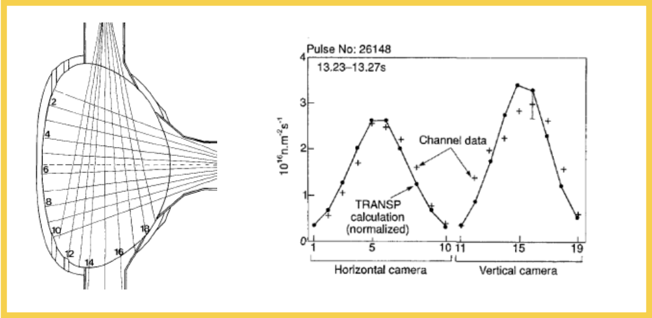
The solution now depends on the regularization parameter λ.

Prior information: The solution is not spiky and definitely not negative

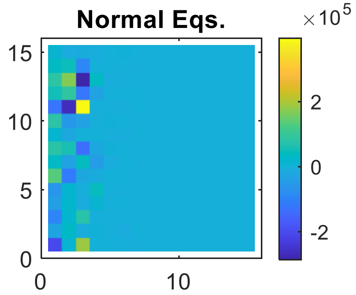
- True solution

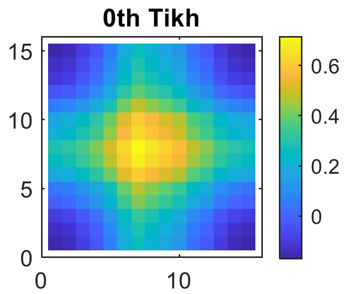






$$F = (W^T W)^{-1} W^T S$$

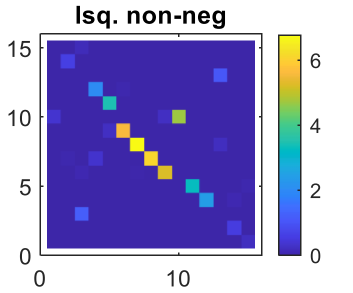


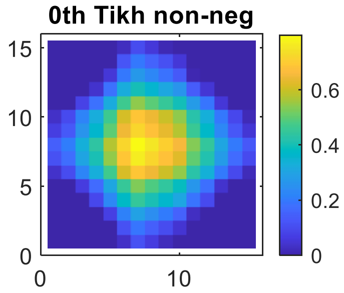


$$F_\lambda = (W^T W + \lambda^2 L^T L)^{-1} W^T S$$

- No non-negativity: distribution function goes negative in the corners

Minimize
$\|WF - S\|_2^2$
subject to $F \geq 0$





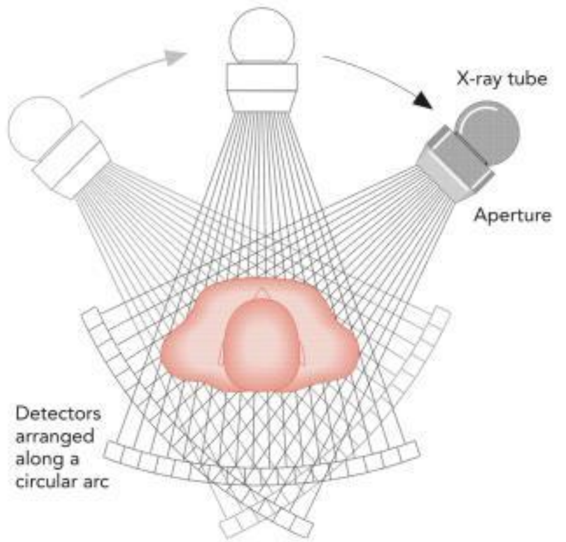
Minimize

$$\left\| \begin{pmatrix} W \\ \lambda L \end{pmatrix} F - \begin{pmatrix} S \\ 0 \end{pmatrix} \right\|_2 \quad \text{subject to} \quad F \geq 0$$

- With non-negativity: distribution function stays positive

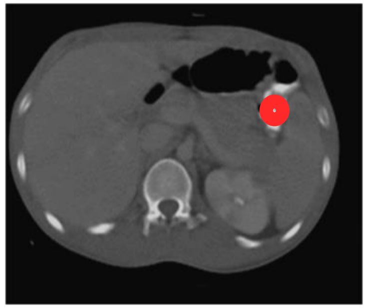
- No regularizaton
- Tikhonov

Tomography in the hospital: CAT scanner

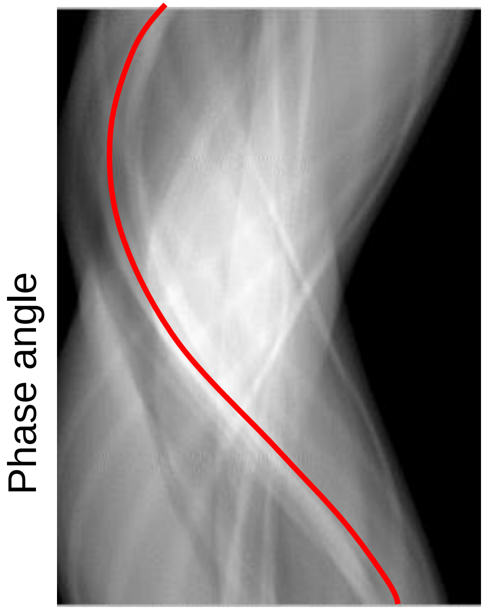


- Red spot in sample traces S-curve in data

Slice through patient

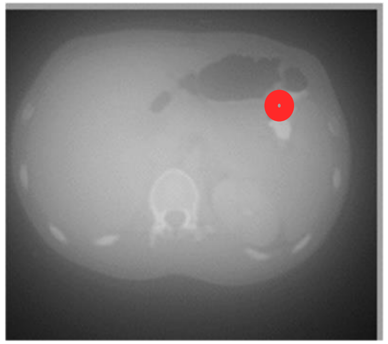


Measurement data



Detector position

Image



Other medical examples:

- PET – positron emission tomography
- MRI – magnetic resonance imaging
- Ultrasound imaging
- Breast mammography
- …

- Cormack 1963, 64
- Hounsfield 1968-73

  Nobel Prize Medicine 1979

Tomography – a forward model for rays at an angle

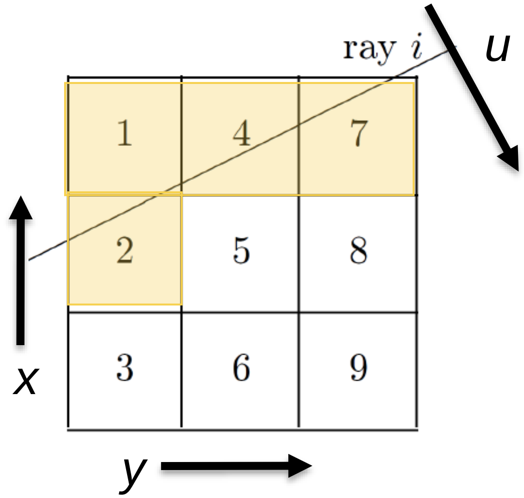
Fredholm integral equation of the first kind:

$$s(u,\phi) = \iint w(x,y,u,\phi) f(x,y)\,dxdy$$

Measured signal depending on detector position *u* and the angle of the rays

Forward model: Given an emitter at (*x*,*y*), signal in detector at *u for a given angle*

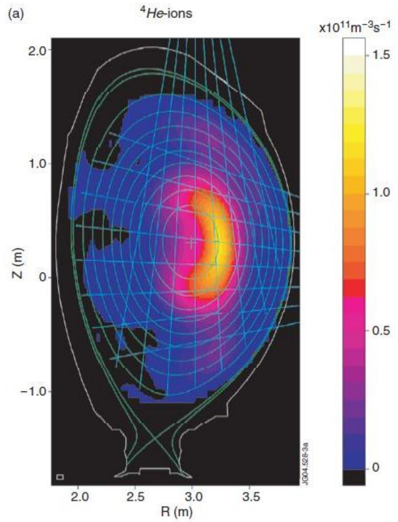
2D distribution of emitters



W_{ij} – length of ray *i* in pixel *j*

Discretize: $S = WF$ $S_i = W_{ij} F_j$

Vector Matrix Vector



Kiptily et al. (2005) NF

Prior information:

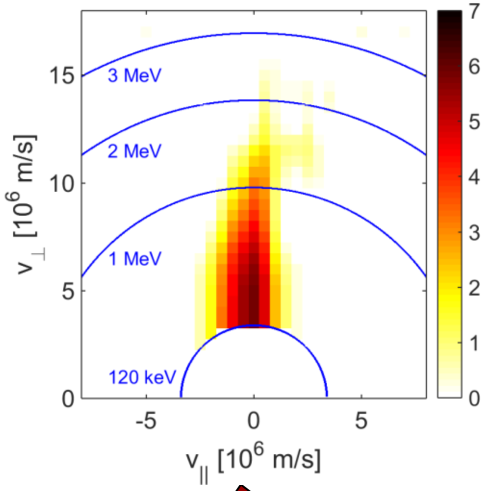
- Non-negativity
- Smoothness
- Magnetic flux surfaces: penalize gradients along flux surfaces more than across

Velocity-space tomography

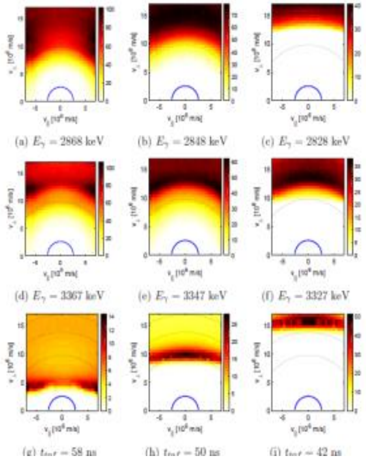
Forward problem

$$WF = S$$

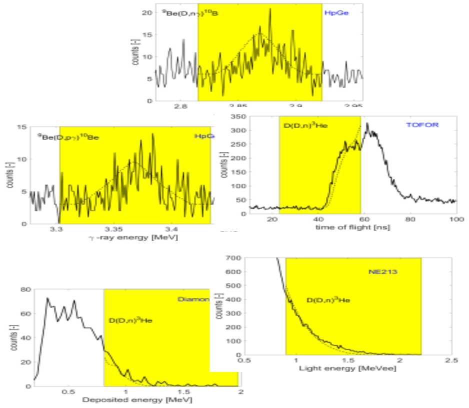
Distribution function F



Matrix W



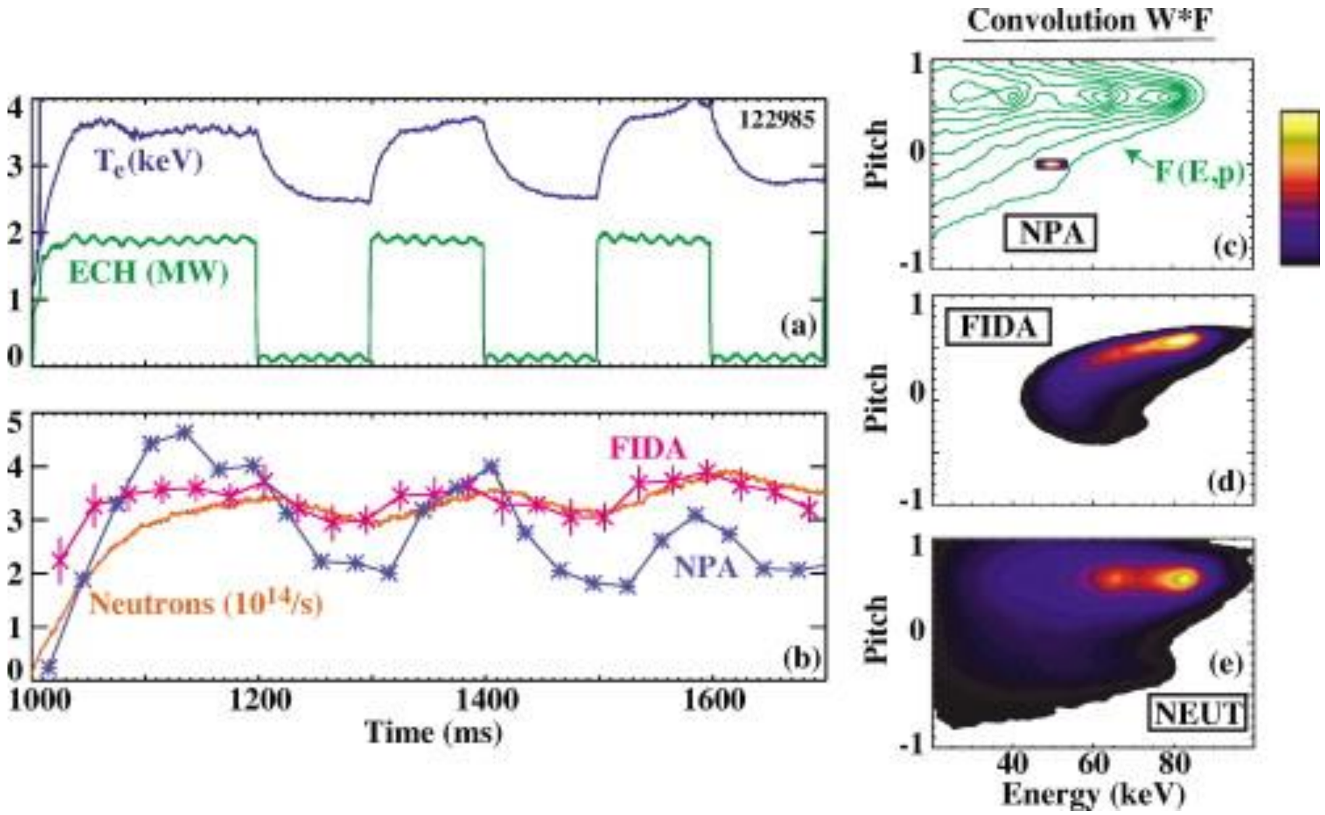
Measurements S



Inverse problem

Salewski 2020, dr. thesis

Velocity-space weight functions



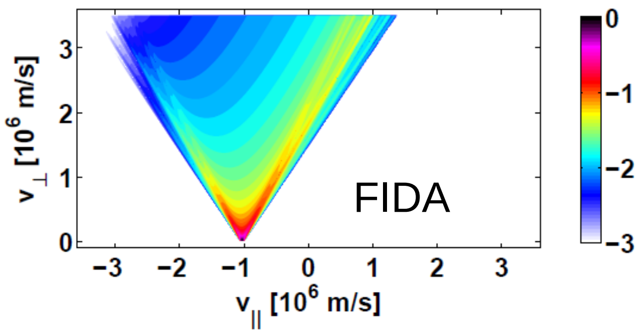
$s = \iint wf\,dEdp$

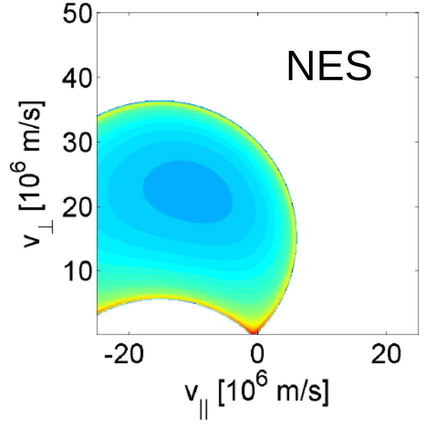
- Signals of FIDA spectroscopy and a neutron counter have similar pattern in time, different from NPA.
- At the time surprising since FIDA and NPA signals come from the same charge-exchange reaction.
- The introduction of weight functions and plotting the integrant *wf* resolved this puzzle.

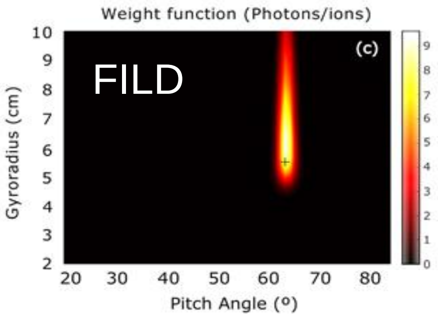
Heidbrink et al. (2007) PPCF, Heidbrink (2010) RSI

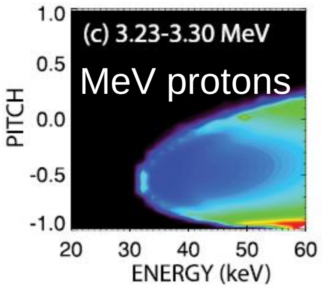
Velocity-space weight functions

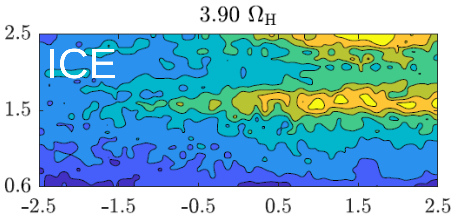
$$s(u_1, u_2, \phi) = \iint w(u_1, u_2, \phi, E, p) f(E, p) dE dp$$

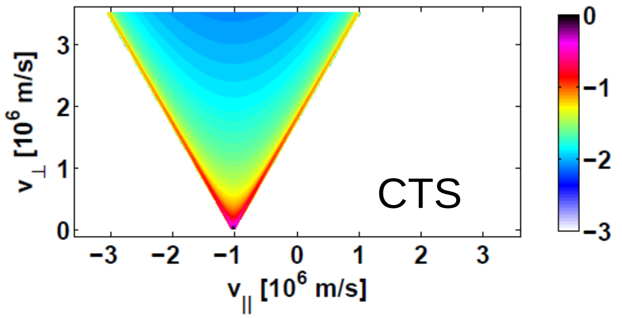


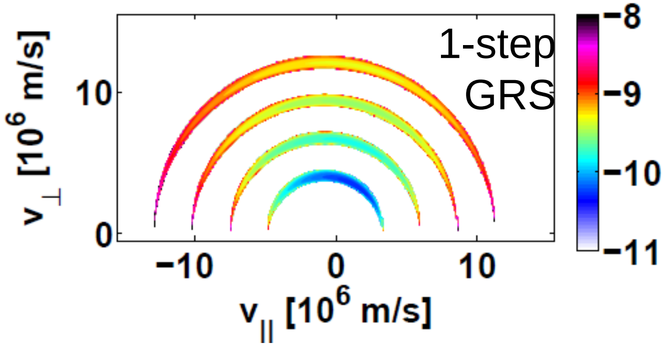


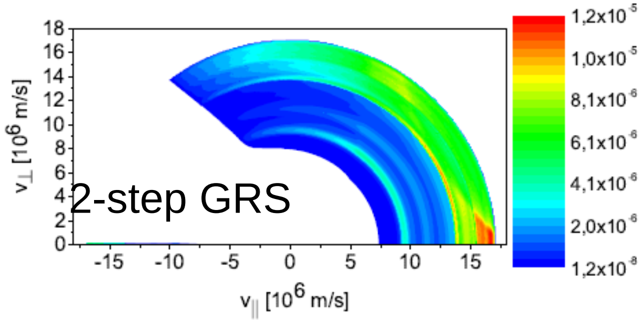


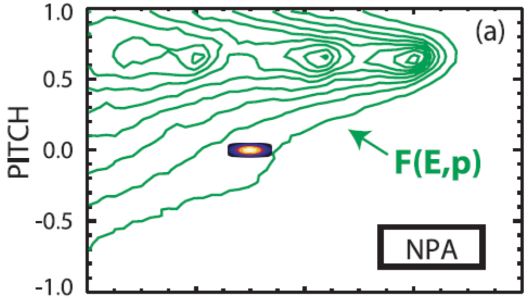












FIDA: Heidbrink et al. (2007) PPCF, Salewski et al. (2014) PPCF

NPA+neutrons+pressue: Heidbrink et al. (2007) PPCF

CTS: Salewski et al. (2011) NF

NES+neutrons: Jacobsen et al. (2015) NF

GRS-2: Salewski et al (2015) NF

GRS-1: Salewski et al (2016) NF

FILD: Galdon-Quiroga et al. (2018) PPCF

MeV protons: Heidbrink et al. (2021) PPCF

ICE: Schmidt et al. (submitted)

Velocity-space weight functions

- The velocity-space weight function w is defined by

$$s(u_1, u_2, \phi) = \iint w(u_1, u_2, \phi, v_\parallel, v_\perp) f(v_\parallel, v_\perp) dv_\parallel dv_\perp$$

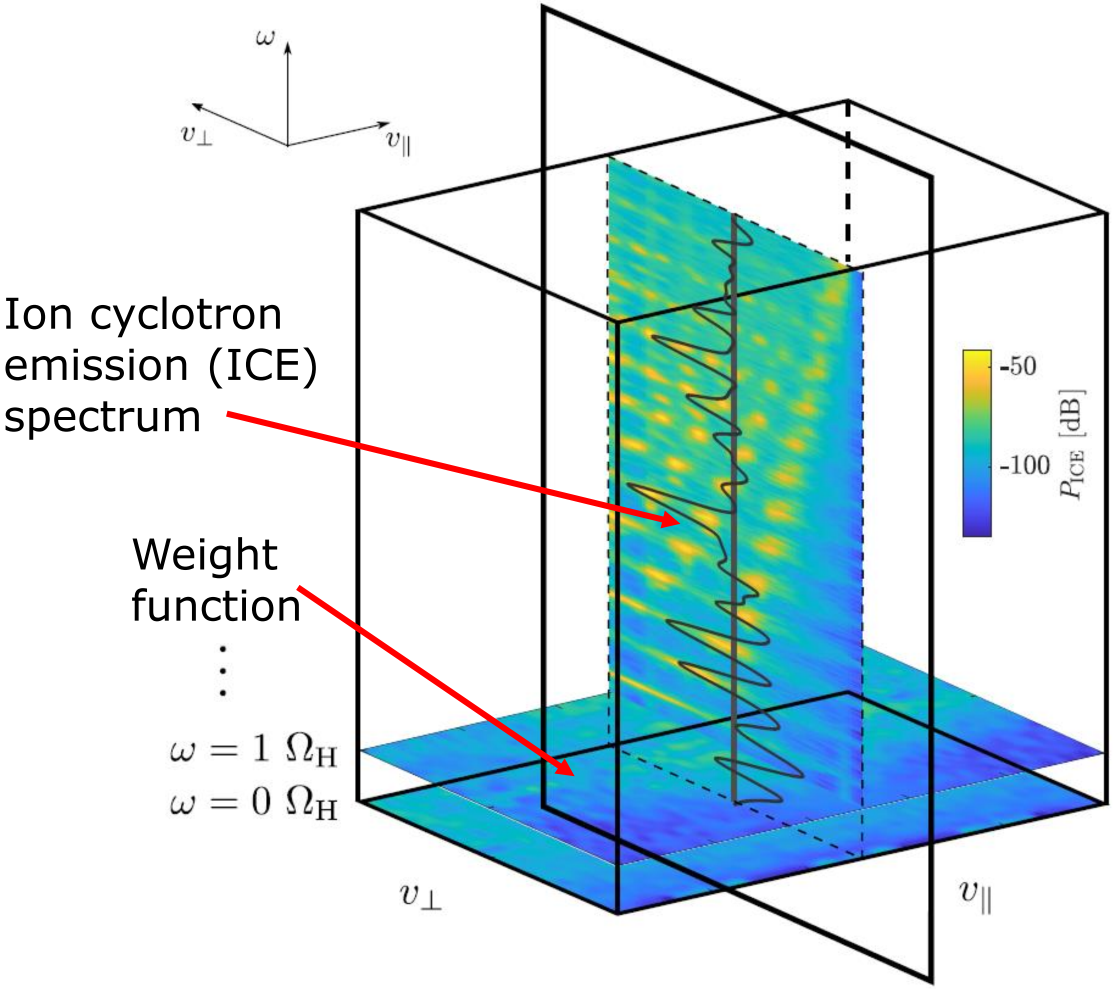
- They are computed by scanning a pixel function through velocity space and computing the signal

$$s(u_1, u_2, \phi) = \iint w(u_1, u_2, \phi, v_\parallel, v_\perp) \delta(v_{\parallel 0}, v_{\perp 0}) dv_\parallel dv_\perp$$

- Effecting the integral gives

$$w(u_1, u_2, \phi, v_{\parallel 0}, v_{\perp 0}) = s(u_1, u_2, \phi)$$

- Practically, compute a spectrum for each point in velocity space and stack them next to each other at their right location in velocity space. Weight functions are horizontal slices.

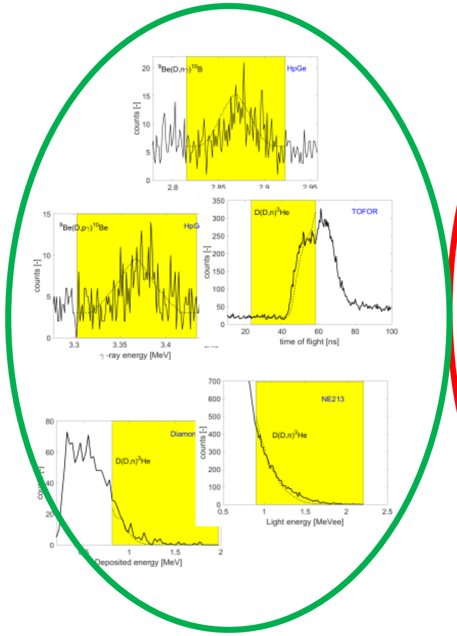


Schmidt (submitted)

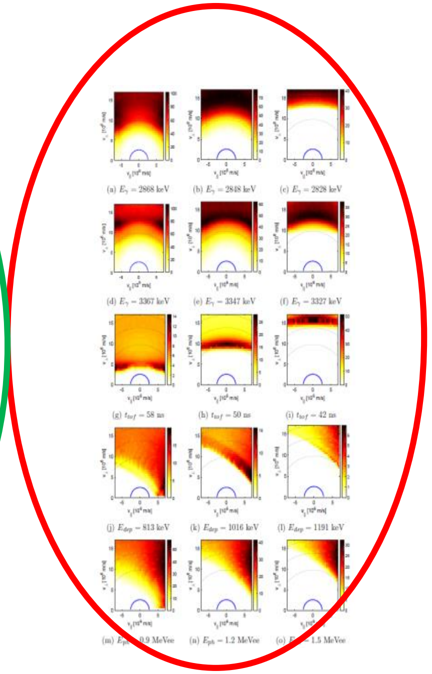
Velocity-space tomography

- Forward problem: $WF^* = S$
- Inverse problem: $\text{minimize} \left\| \begin{pmatrix} W \\ \lambda L \end{pmatrix} F - \begin{pmatrix} S \\ 0 \end{pmatrix} \right\|_2 \quad \text{subject to} \quad F \geq 0$

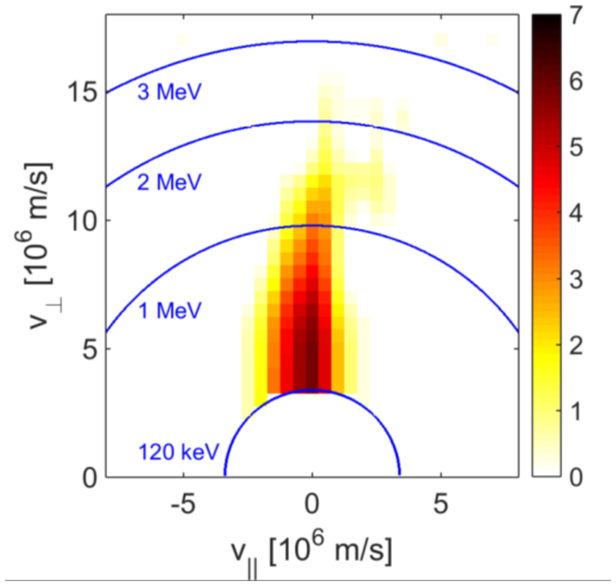
Measurements



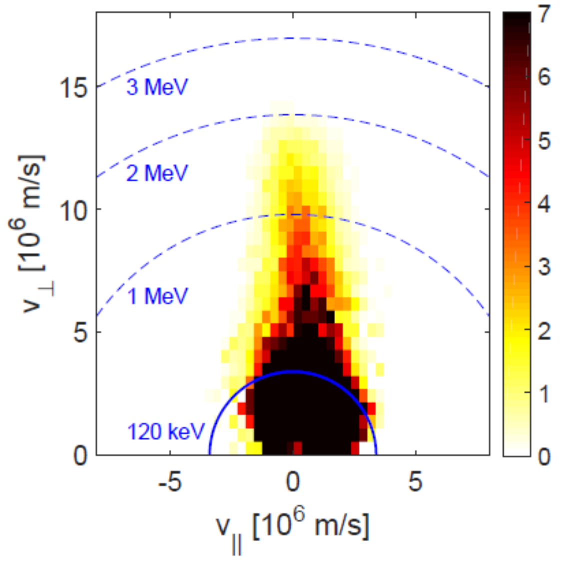
Weight functions



Measured velocity distribution function



ASCOT simulation

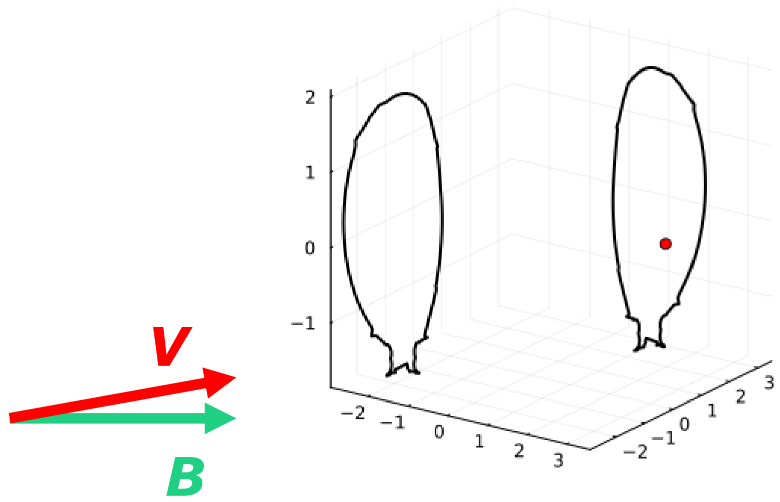


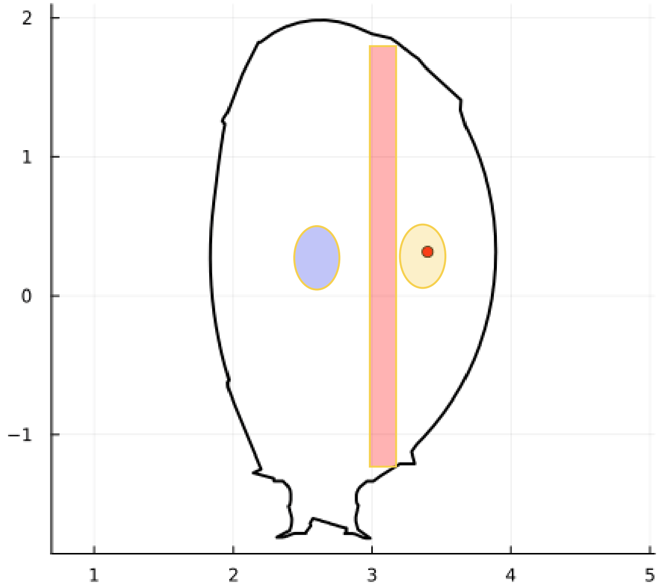
Prior information: smoothness, non-negativity, null-measurements, beam positions, numerical simulation, near-isotropy

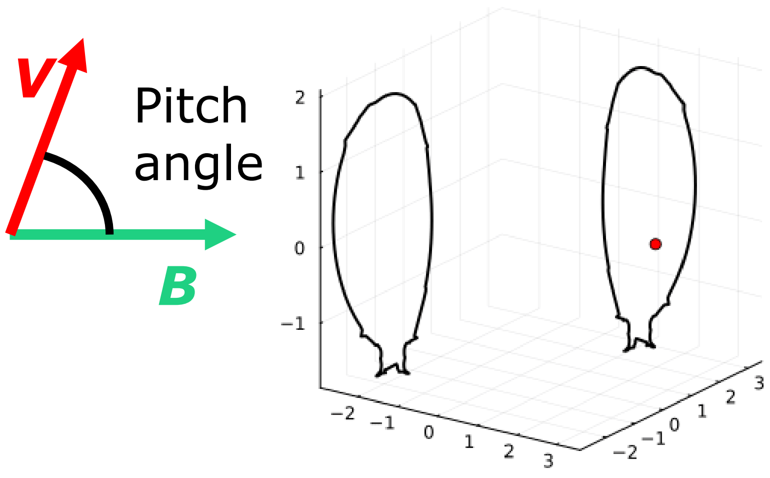
Current work: Collision physics as prior information. Slowing-down distribution functions reflect the physics of collisions in fusion plasmas. Strong prior in 2D to 5D phase-space tomography *(Madsen et al. (2020) PPCF, Schmidt et al. (2023) NF)*

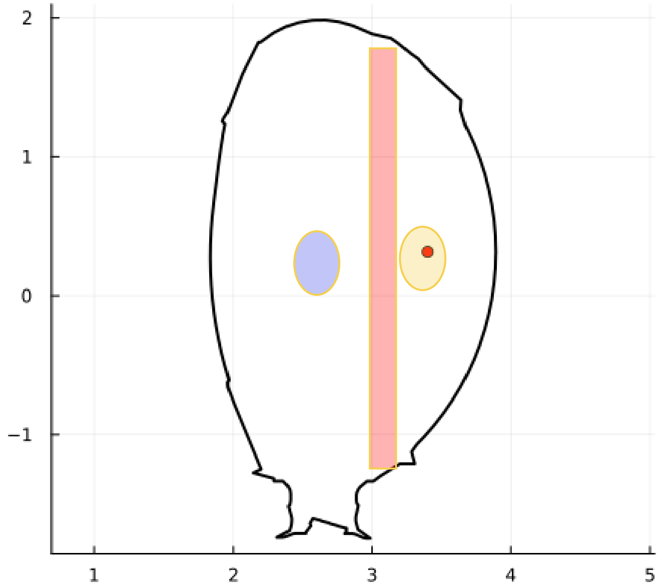
Orbit tomography

- Energetic particles trace out surfaces on their drift orbits, e.g. a passing particle and a trapped (banana) particle
- All ions in a tokamak plasma are completely described by a 3D phase space distribution function $f(E, \mu, P_\Phi, \sigma)$ (energy, magnetic moment, canonical toroidal angular momentum)
- Each point in this space corresponds to a drift orbit
- Orbit tomography: find $f(E, \mu, P_\Phi, \sigma)$ from measurements.
- Red: Line-of-sight, blue and yellow: measurement volume







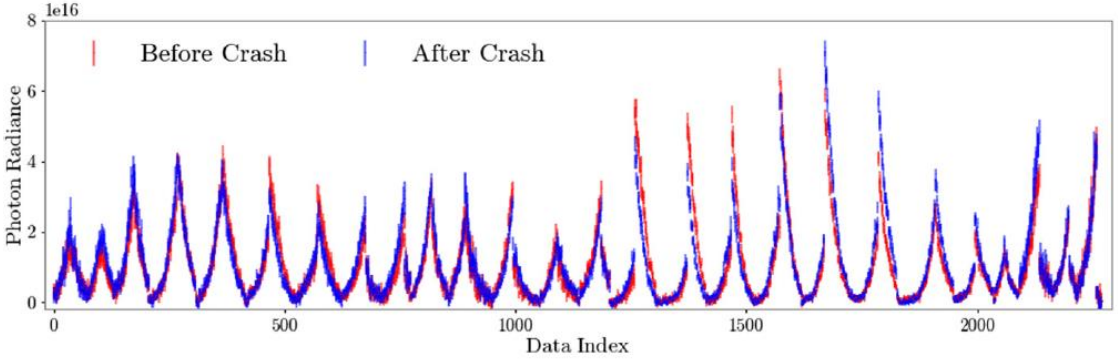


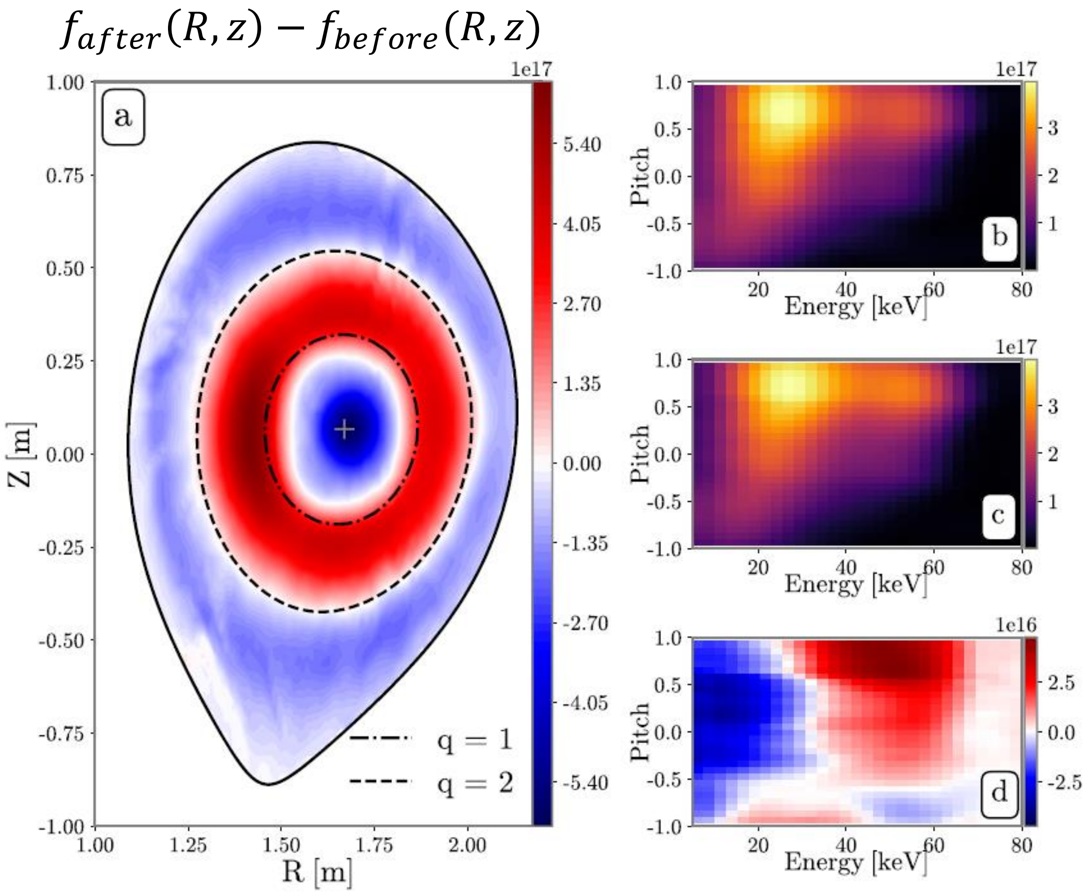
Järleblad

Orbit tomography

• Orbit tomography before and after a sawtooth crash shows ejection of particles from the plasma core at ASDEX Upgrade

• Requires many simultaneous spectra with good spread in projection angle (tangential to vertical view), here 27 fast-ion D-alpha spectra.





Current work: Collision physics as prior information. Slowing-down distribution functions reflect the physics of collisions in fusion plasmas. Strong prior in 2D to 5D phase-space tomography *(Madsen et al. (2020) PPCF, Schmidt et al. (2023) NF)*

Stagner et al. (2022) NF

Discussion: Energetic particle diagnostics at ITER

Original PFPO-2 Baseline

Pre-fusion power operation with 53 MW NBI+ICRF

55.E8 Neutral Particle Analyzer *(perpendicular, radial)*

This list refers to the original ITER baseline which is currently under revision.

Original FPO Baseline

Fusion power operation

55.E8 Neutral Particle Analyzer *(perpendicular, radial)*

55.B1 Radial Neutron Camera *(perpendicular, radial)*

55.B2 Vertical Neutron Camera *(perpendicular, vertical)*

55.BV Neutron Calibration

55.C7 Collective Thomson Scattering (back-end not yet baseline) *(perpendicular)*

Not yet baseline

55.B7 Radial Gamma Ray Spectrometer *(perpendicular, radial)*

55.B9 Lost Alpha Monitor *(perpendicular, radial)*

55.BB High Resolution Neutron Spectrometer *(perpendicular, radial)*

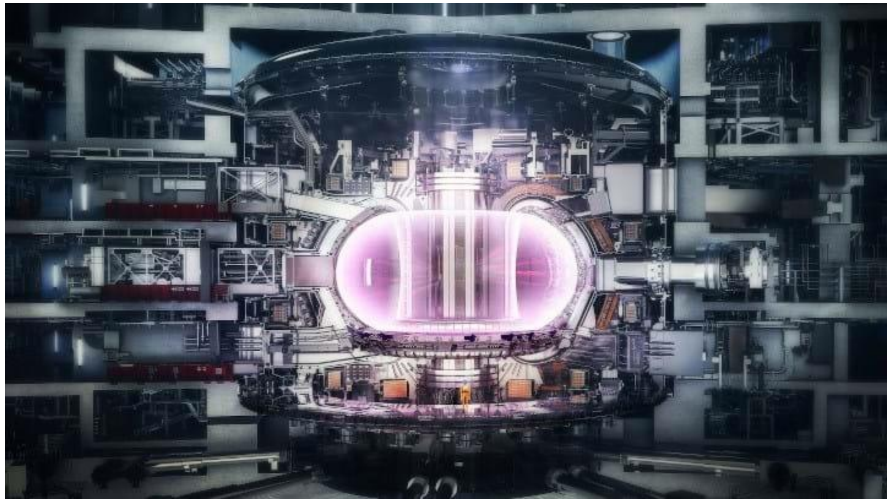
55.BD Vertical Gamma Ray Spectrometer *(perpendicular, vertical)*

55.BE Tangential Neutron Spectrometer *(oblique)*

Additionally, work is being done on fast-ion loss detectors and ion cyclotron emission spectroscopy but they have not received official designations.

Discussion: Energetic particle diagnostics at ITER

Discussion: How would *you* design energetic particle diagnostic system for a burning plasma experiment?

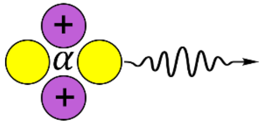


1) What would *you* like to measure?

2) Which diagnostics would *you* use to accomplish this?

3) How would *you* arrange them?

4) Compare to the diagnostic set at ITER? What compromises have been made?



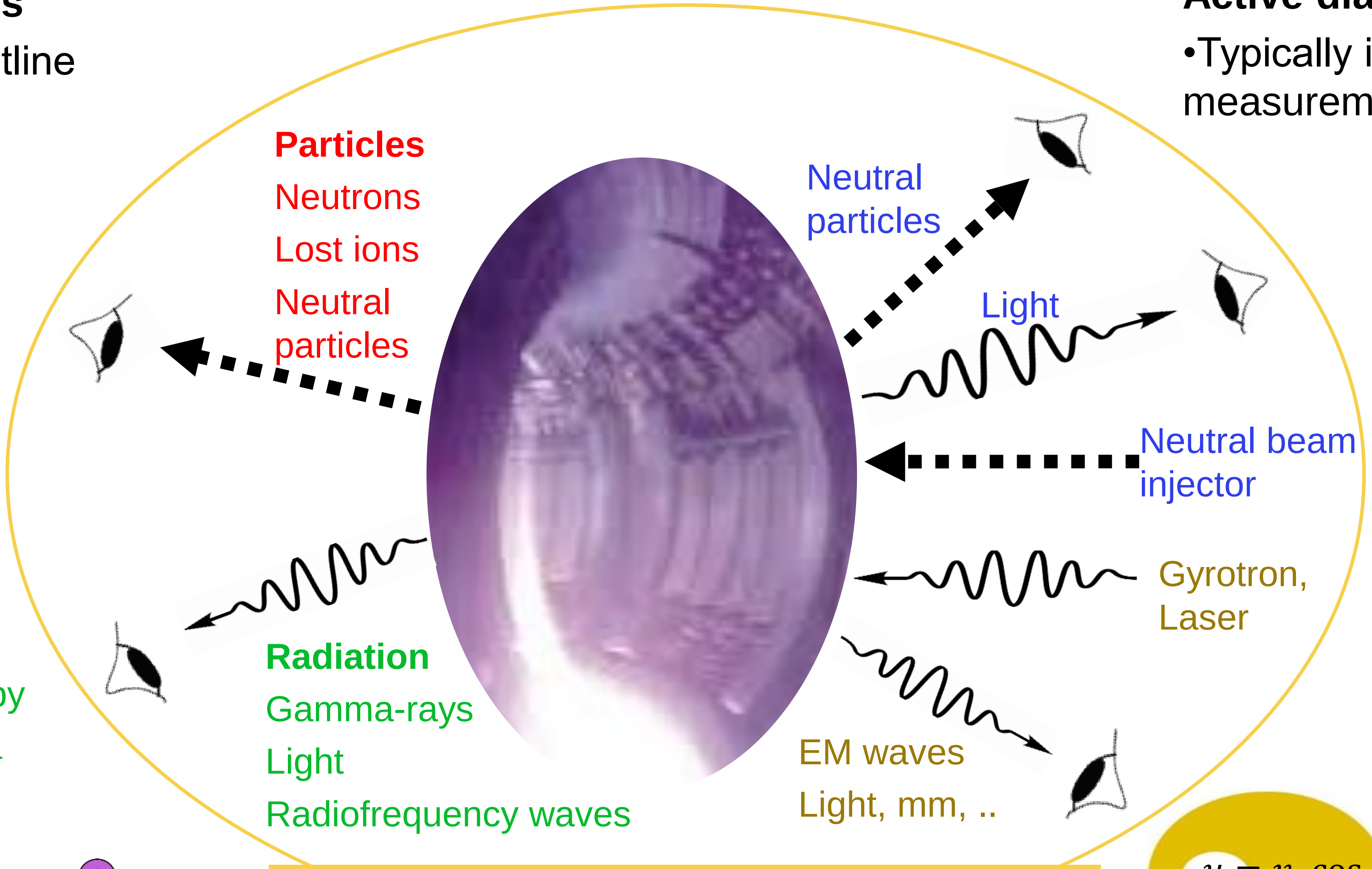
Summary of energetic particle diagnostics

Passive diagnostics

•Typically along sightline

- •Neutron counter
- •Neutron camera
- •Neutron emission spectroscopy
- •Fast-ion loss detector
- •Charged fusion product detector
- •Passive neutral particle analyser

- •Gamma-ray camera,
- •Gamma-ray spectroscopy
- •Passive fast-ion D-alpha spectroscopy
- •Ion cyclotron emission spectroscopy

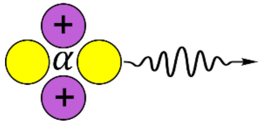


Active diagnostics

•Typically in a small measurement volume

- •Neutral particle analyser
- •Imaging neutral particle analyzer
- •Fast-ion D-alpha spectroscopy

- •Collective Thomson scattering



Integrated data analysis of available diagnostics by 1D-5D phase-space tomography!

$$u = v_\parallel \cos\phi + v_\perp \sin\phi \cos\Gamma$$